MD IFTAKHAR KABIR SAKUR

25th BATCH

COMPUTER AND COMMUNICATION ENGINEERING

International Islamic University Chittagong

COURSE CODE: CCE-3503

COURSE TITLE: Information Theory and Error Coding

COURSE TEACHER:

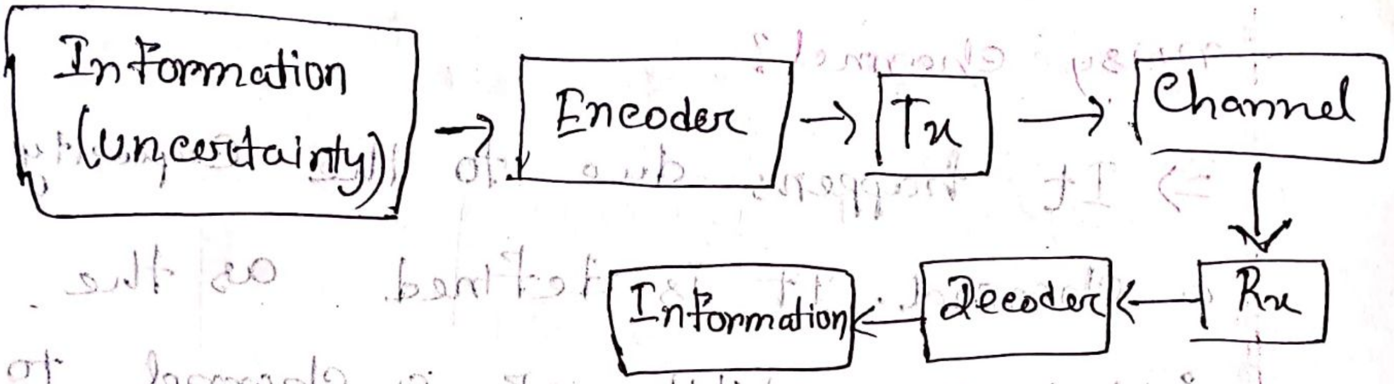
PROF. RAZU AHMED

&

Md Humayun Kabir

Book: Communication System
(Symon Hykin)

Chapter - 9 (Midterm - Exam)



(অপরিবর্তনীয়)

㊟ what is the irreducible Complexity below which a signal can't be Compressed?

⇒ This ~~due~~ happens due to entropy of a source.

The entropy is defined in terms of the probabilistic behavior of a source of information. It is so named in deference to the parallel use of ~~its ther~~ this Concept in Thermodynamics.

⊛ What is the ultimate transmission rate for reliable communication over a noisy channel?

=> It happens due to the capacity of a channel. It is defined as the intrinsic ability of a channel to convey information. It is naturally related to the noise characteristics of the channel.

Note:-

A remarkable result that emerges from information theory is that if the entropy of the source is less than the capacity of channel, then the error free communication over the channel can be achieved.

It is therefore befitting that we begin our study of information theory by discussing the relationships among uncertainty, information & entropy.

Uncertainty

প্রতি একক সময়ে ~~ভিন্ন ভিন্ন~~ একটি পৃথক ~~পরীক্ষা~~ উৎস হতে পাওয়া আউটপুট যা সম্ভাব্য পরীক্ষার পর্যবেক্ষণকে জড়িত করে।

এর উৎস আউটপুট কে পৃথক বিচ্ছিন্ন ~~কি~~ বিচ্ছিন্ন পরিবর্তনশীল হিসেবে মডেল বা গণ্য করা হয়।

The variable is S. which takes on symbols from a fixed finite alphabet,

$$S = \{s_0, s_1, \cdots s_{K-1}\}$$

with probabilities,

$$P\{S = s_K\} = P_K$$

$$K = 0, 1, \ldots, K-1$$

$$\sum_{K=0}^{K-1} P_K = 1$$ ~~{ total~~

=)

$$x = \{x_0, x_1, x_2, \ldots, x_n\}$$

$$P = \{P_0, P_1, \ldots, P_n\}$$

Total probability, $P = \sum_{i=1}^{m} P_i$

$$\left.\begin{array}{l} x_0 \rightarrow P_0 = 0 \\ \qquad P_1 = 1 \end{array}\right\}$$ NO uncertainty

$x_0 \rightarrow P_0 = 0.5$ ⊛ probability↓, Uncertainty↑

$P_1 = 0.1$

◻ Measure of information :-

It is a information constant of a message

⇒ Consider an information source emitting (নিক্ষেপ করা) independent messages.

$m = \{m_1, m_2, \ldots m_n\}$ with probability of occurrence is,

$P = \{P_1, P_2, + \ldots, P_n\}$.

Hence, $P_1 + P_2 - - - - + P_{n-1}$

Amount of information is,

$$I_K = \log_2 \left(1/P_K\right) = \frac{\log_2 (1/P_K)}{\log_2}$$

$$\text{i.e. } I_1 = \log (1/P_1) = \log_2 \left(\frac{1}{1/4}\right) = 2 \log_2 2 = 2$$

Important Topics on Entropy

We define the amount of information gained after observing the event $S = s_K$, which occurs with probability p_K, as the logarithmic function

$$I(s_K) = \log\left(\frac{1}{p_K}\right)$$

① ⇒ $I(s_K) = 0$ for $p_K = 1$

(If we are absolutely certain (निश्चित) of the outcome of an event, even before it occurs, there is no information gained)

② ⇒ $I(s_K) \geq 0$ for $0 \leq p_K \leq 1$

(The occurrence of an event $s = s_K$ either provides some or no information but never brings about a loss of information.)

③ $I(s_K) > I(s_i)$ for $p_K < p_i$

⇒ That is, the less probable an event is, the more information we gain when it occurs.

④ $I(s_K s_i) = I(s_K) + I(s_i)$

⇒ If s_K & s_i are statistically independent.

⊛ $I(s_K) = \log_2(1/p_K)$

$= -\log_2 p_K$ for, $K = 0, 1, \ldots K-1$

The resulting unit of information is called the bit. when $p_K = 1/2$ we have $I(s_K) = 1$ bit.

⊛ $I(s_K)$ is a discrete random variable that takes on the values $I(s_0), I(s_1), \ldots$ & $I(s_{K-1})$ with probabilities $p_0, p_1, \ldots p_{K-1}$ respectively.

Math

⊛ Receiver knows message being transmitt
then information is 2000

$\Rightarrow \quad P_K = 1$

$$I = \log_2 (P_K)$$

$$= \log_2 (1)$$

$$= 0$$

⊛ ~~I~~ I_1 is the information of message m_1 & I_2 of m_2 then show that combined information of m_1 & m_2 = $I_1 + I_2$

$\Rightarrow \quad I_1 = \log_2 \left(1/P_1 \right)$

$I_2 = \log_2 \left(1/P_2 \right)$

$$I = \log_2 \left(1/P \right) = \log_2 \left(\frac{1}{P_1 P_2} \right)$$

$$= \log_2 \left(1/P_1 \right) + \log_2 \left(1/P_2 \right)$$

$= I_1 + I_2$ [As ~~Information~~ message m_1 & m_2 are independent so combined probability $P_1 P_2$]

Q. ⊛ If there are $m = 2^N$ equally likely message the amount of information carried by each message will be lots ~~of~~ of probabilities of message, $P = 1/m$

$$\Rightarrow I = \log_2 (1/p) = \log_2 \left(\frac{1}{1/m} \right)$$

$$= \log_2 (m)$$

$$= \log_2 2^N$$

$$= N \log_2 2 = N \text{ bits}$$

~~⊛ Calculate the probability of information~~

~~For two messages where~~

⊛ Calculate the amount of information for two messages where probability,

$P_1 = 1/4$ & $P_2 = 3/4$

$$I_1 = \log_2(1/P_1) = \log_2(4) = \log_2 2^2$$

$$= 2$$

$$I_2 = \log_2(1/P_2) = \log_2(4/3)$$

$$I = \left(\log_2(1/P)\right)$$

$$= \log_2\left(\frac{1}{P_1 \cdot P_2}\right)$$

$$= \log_2(1/P_1) + \log_2(1/P_2)$$

$$= 2 + \log_2(4/3)$$

= ~~lo~~ $2 + 2 - \log_2(1/3)$

$$= 4 - \log_2(1/3)$$

52 ㉖ ×26

$P = 1/2$

$I = \log_2 1/x$ $1/2$

❀ A card is selected at random from a deck of playing cards & you have been told that it is red colour.

→ How much information you have received?

⇒ We know,

There are 52 cards

26 are red

probability, $P = \frac{26}{52} = \frac{1}{2}$

$$\therefore I = \log_2\left(\frac{1}{1/2}\right) = \log_2 2$$

$$= 1 \text{ bit}$$

Entropy ✓

It is average information of symbols if we have $m = \{x_1, x_2, \dots x_n\}$ messages with probability $P = \{P_1, P_2, \dots P_n\}$ then information of each messages

$$I_1 = \log_2 (1/P_1), \quad I_2 = \log_2 (1/P_2), \quad I_3 = \log_3 (1/P_3)$$

$$\dots \quad I_n = \log_2 (1/P_n)$$

$$\therefore \text{Entropy}, \ H = \frac{\text{Total Information}}{\text{No. of messages}}$$

$$= \frac{I_1 + I_2 + I_3 + \dots + I_n}{n}$$

$$\text{entropy}, \ H = \sum_{i=1}^{n} P_i \log_2 (1/P_i) \text{ bits / symbols}$$

properties

⊛ Entropy is zero if the event is same,

$\begin{cases} P_{.} = 0 \\ P = 1. \end{cases}$

$\Rightarrow P = 0$

$$H = \sum_{k=1}^{m} p_k \log_2 \left(1/p_k \right)$$

$$= \sum_{i=1}^{m} 0 . \log_2 \left(1/0 \right)$$

$$= 0$$

if $P = 1$

$$H = \sum_{i=1}^{m} p_k \log_2 \left(1/p_k \right)$$

$$= \sum^{m} 1 . \log_2 \left(1/1 \right) = 0$$

② when, $P = 1/m$ for all "m" symbols then symbols are equal

So, $H = \log_2 m$

$\Rightarrow P_K = 1/m$

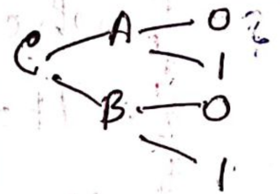
$$H = \sum_{K=1}^{m} 1/m \, \log_2 \left(\frac{1}{1/m} \right)$$

$$H = \log_2 m = H_{max}$$

problem:-

Consider discrete memoryless Source (DMS) that output two bits at a time. This source comprises two binary sources 'A' & 'B' where output are equally likely to occur & each source contributing one bit. Suppose that the sources within the source ε) are independent. what is the information content of each output from C.

Understanding



➡

Total symbol of C

$C = \{00, 01, 10, 11\}$

$P_C = 1/4$

or,

$P_A = 1/2$ & $P_B = 1/2$

Combined probability of A & B

$P_C = P_A \cdot P_B$

$= \frac{1}{2} \cdot \frac{1}{2} = 1/4$

Information of C, $I_C = \log_2\left(\frac{1}{1/4}\right)$

$= \log_2(4)$

$= \log_2 2^2$

$= 2$ bits

Source of Efficiency

$$\eta = \frac{H\ (\text{Entropy})}{H_{max}}$$

where,

H = Calculated Entropy of source

H_{max} = Maximum Entropy

$H_{max} = \log_2 m$

(অপ্রয়োজনীয়তা)

Redundance of source,

$R_c = 1 - \eta$

Information Rate

$R = r H$ | r = rate at which messages are generated (Message/sec)

~~=bi~~

Unit :- bits/sec

Math

⊛ The source emits 3 messages with probability,

$P_1 = 0.7$

$P_2 = 0.2$

$P_3 = 0.1$

Calculate: (i) Source of entropy

(ii) Maximum Entropy

(iii) Source Efficiency

(iv) Redundancy

Solution:–

(i) Source of Entropy,

$$H = \sum_{i=1}^{3} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= 0.7 \log_2\left(\frac{1}{0.7}\right) + (0.2) \log_2\left(\frac{1}{0.2}\right) + 0.1 \log_2\left(\frac{1}{0.1}\right)$$

Calculator এ হিসাব করতে গেলে Base ওয়ালা log দিয়ে করতে হবে।

$= \cancel{0.3482}$ ~~bits/message~~

$= 1.1568$ bits/message

(2) Maximum Entropy,

$$H_{max} = \log_2 M$$

$$= \log_2 3$$

$$= 1.585 \text{ bits/message}$$

(3) Source of efficiency,

$$\eta = \frac{H}{H_{max}} = \frac{1.1568}{1.585} = 0.7299$$

(4) Redundancy,

$\cancel{1} -$

$$R_e = 1 - \eta = 1 - 0.7299$$

$$= 0.27$$

(Ans)

Discrete Memoryless Source (DMS):-

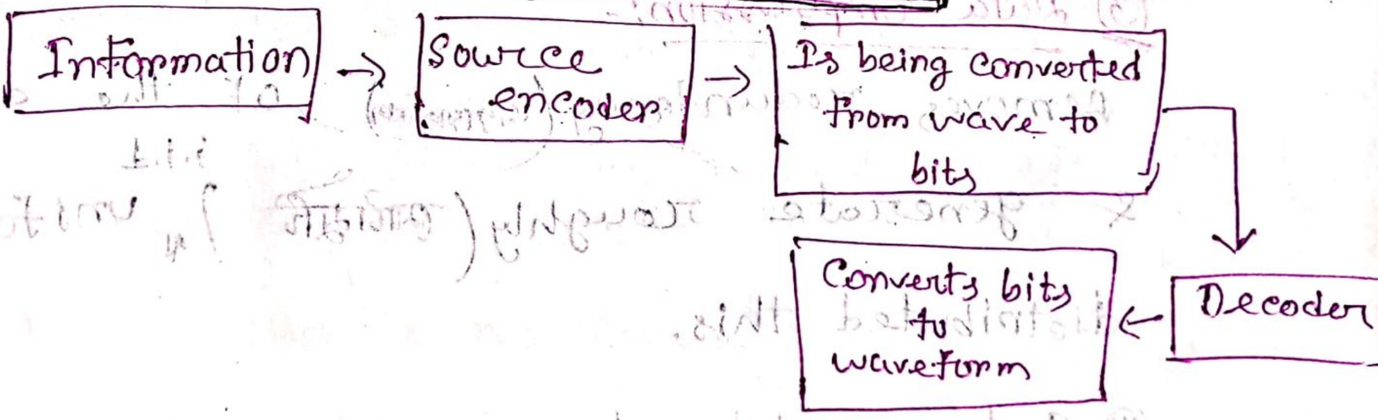
A source from which data is being emitted at successive intervals, which is independent of previous values, can be termed as discrete memoryless source.

It is not concidered for a continuous time interval, but at discrete time intervals.

This source is memoryless as it is fresh at each instant of time, without considering the previous values.

IMP

Source Coding



→ Source encoding aims to convert information waveforms (Text, Audio, video, image etc) into bits, the universal currency of information in the digital world.

3 major steps:-

① **Sampling:-**

Converts,

Continuous-time Analog waveform → Discrete-Time Sequence

(But still remains in continuous valued)

② **Quantization:-**

Converts,

Continuous valued Symbols → Discrete valued Representatives

③ **Data Compression:-**

Removes redundancy (অপ্রয়োজনীয়তা) of the data & generate roughly (মোটামুটি) i.i.d uniformly distributed this.

⊛ Code-word is always greater than or equal to the source-code. which means the symbols in the code word are greater than or equal to the alphabets in the source code.

Channel Coding:-

It introduces redundancy with a control in a communication system. So, as to improve the reliability of the system. It improves the efficiency of the system.

It has two parts:-

Mapping:- Incoming data sequence into a channel input sequence. It happens in transmitter, with the help of an encoder.

~~Inverse Mapping~~:-

Inverse Mapping:-

The channel output sequence into an output data sequence. It happens in the receiver by a decoder.

The target of it is that the overall effect of the channel noise should be minimized.

Channel Coding:-

The channel encoder converts bits to signal waveform, while the decoder converts received waveform back to bits.

There are four major steps of it:-

① **Error correcting codes:-**

Introduce redundancy into the information bits and produce longer coded bits.

Example:- Repeatition Code: each bit repeat N times

Channel noise:-

Flip the bit n.p. p, w.p. $(1-p)$ remain the same.

- Bit error happens when there are more than $N/2$ bit flips:

$$Pr(error) = 1 - \sum \binom{N}{i} p^i (1-p)^{n-i}$$

→ Symbol Mapping:-

Map the coded bits to constellation points, each of which is a complex symbol.

→ pulse shaping:-

Modulate the symbol to suitable baseband waveforms. There are some specific conditions needed to be satisfied.,

→ ~~UP Conversation:~~

→ UP Conversion:-

Converts the baseband waveform to passband waveform, so that the effective frequency band follows the constraints (সীমাবদ্ধতা) from the physical world.

Lecture - 03

⊛ A system of measurement of information based on the probabilities of the events that convey information is called as measure of information.

⊛ লগারিদমিক ভিত্তির উপর নির্ভর করে ইনফরমেশন মাপা যায়। এক্ষেত্রে একক use করা হয়।

→ bit (binary logarithm)

→ nat (natural logarithm)

→ hartley (based on the base 10)

→ common logarithm.

Natural Logarithmic base : nat

Base-10 : Hartley / decit

Base-2 : bit

⊛ If probability,

$P_1 = P_2 = 1/2$ then the correct identification of the binary digit conveys an amount of information.

$I(m_1) = I(m_2) = -\log_2(1/2) = 1$ bit

Example:-

$P_1 = 1/2$

$I(m_1) = -\log_2(1/2)$

$= 1$ bit

$P_2 = 1/4$

$I(m_2) = -\log_2(1/4) = -\log_2(2^{-2})$

$= 2\log_2 2 = 2$ bits

$I(m_3) = -\log_2(1/8) = -3$ bits

$I(m_4) = -\log_2(1/16) = 4$ bits

$I(m_5) = -\log_2(1/16) = 4$ bits

Self Information :-

Shannon derived a measure of information content called the ~~st~~ self information or "surprisal" of a message m:-

$$I(m) = \log\left(\frac{1}{P(m)}\right) = -\log(P(m))$$

Here, $P(m) = pr(M=m)$ it's the probability that message m is chosen in the message space M.

If the logarithm is base 2, the measure of information is expressed in unit of bits.

Information Rate:-

$R = r_s \cdot H$ bits/sec.

~~R = 1~~

R = Info Rate

~~r_s = Sympo~~

symbols at a fixed rate; r_s = symbols/sec.

Math

An analog signal is band limited to B Hz, sampled at the Nyquist rate, and samples are quantized into 4 levels. The quantization levels Q_1, Q_2, Q_3 & Q_4 (message) are assumed independent and occur with probabilities.

⇒ $P_1 = P_4 = 1/8$ & $P_2 = P_3 = 3/8$. Find the info. rate of the source.

Solution:-

The average information H is given by,

$$H = P_1 \cdot \log 1/P_1 + P_2 \log 1/P_2 + P_3 \log 1/P_3 + P_4 \log \frac{1}{P_4}$$

Math:-

☐ A black & white TV picture Consists of 525 lines of pictures elements, each elements having 256 brightness levels & the pictures are repeated at the rate of 30 F.P.S. Calculate the average info Converged by a TV set to a viewer.

⇒ one picture contains 525 lines

one picture line contains 525 picture elements.

pictures are set at 30 FPS.

Picture element rate, $r = \cancel{256 \times 2}$

$= 525 \times 525 \times 30$

$= 8.26875 \times 10^6$ pictures element/sec

$H = \log_2 M$

Here, $M = 256$ brightness levels.

$\therefore H = \log_2 256 = 8$ bits

$\therefore R = r \cdot H$

$= 8.26875 \times 10^6 \times 8$

$= 66.15$ Mbps.

Shannon's Source Coding Theorem

The Fundamental problem of Communication is that of reproducing at one point either exactly or approximately a message selected at another point.

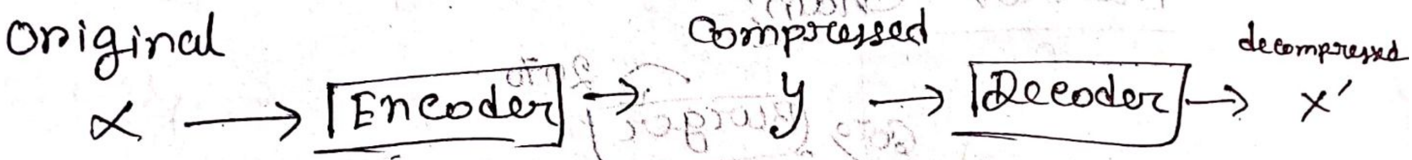
Modem → Line + noise → Modem

That means our only problem is the noise!

⊛ Expected code word length is the entropy of x

$E(C(x) = H(x)$

The noiseless coding Theorem:-



Loseless compression:- $x = x'$

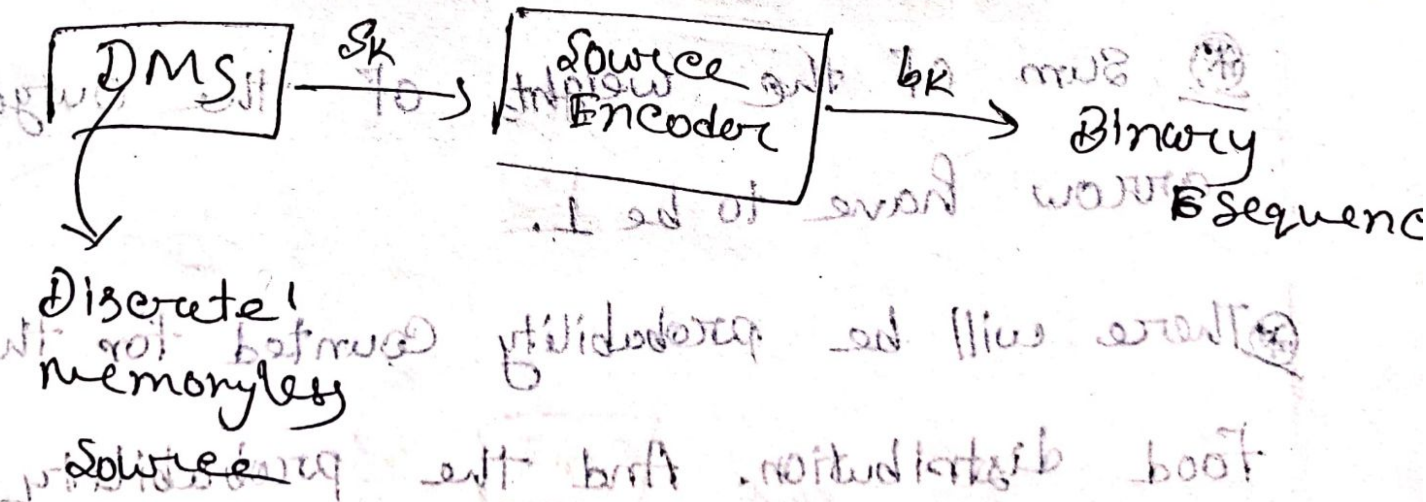
App:- text compression

"Do not send money" → "Do now send money"

Lossey compression:- $x \neq x'$

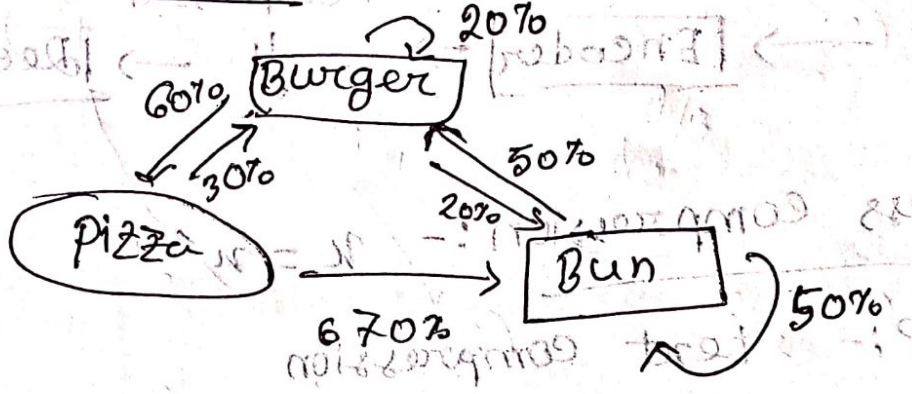
App:- image compression

keyword: distortion vs perception.



Markov Source

Markov chain:-



⊛ The Future state only depends on the current state,

Mathmatically,

$$P(x_{n+1} = x \mid x_1 = x_1 \cdot x_2 = x_2, \dots x_n = x_n)$$

⊛ Sum of the weights of the outgoing arrow have to be 1.

⊛ There will be probability counted for the Food distribution. And the probability will not change for Markov Chain.

Math

Consider the Markov chain with three states, $S = \{1, 2, 3\}$ that has the following transition matrix.

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

(a) Draw the state transition diagram for this chain.

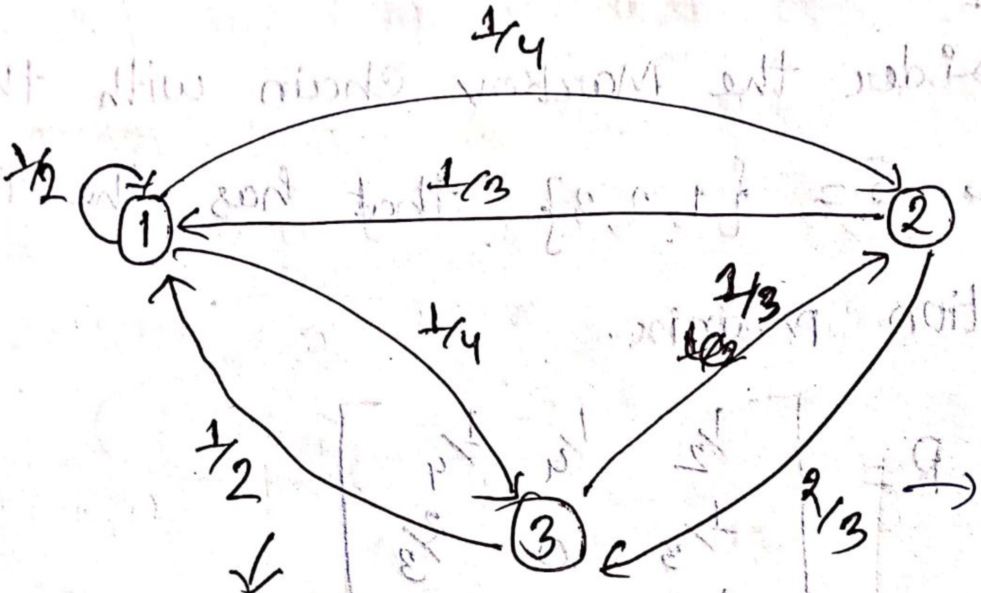
(b) If we know $P(X_1 = 1) = P(x_1 = 2) = 1/4$

Find $P(x_1 = 3, x_2 = 2, x_3 = 1)$

Solve

(a)

P.T.O



→ 2 এর সাথে
3 এর সম্পর্ক

↓
3 এর সাথে
1 এর সম্পর্ক

$$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline & 1/2 & 1/4 & 1/4 \\ & 1/3 & 0 & 2/3 \\ & 1/2 & 1/2 & 0 \end{array}$$

→ 1 এর সম্পর্ক
→ 2 এর "
→ 3 এর সম্পর্ক

(b)

1/4 1/4 1/2

Ans:-

Joint Entropy:-

The Joint entropy represents the amount of information needed on average to specify the value of two discrete random variable.

Conditional Entropy

Given a pair of random variable (x,y) the conditional entropy H(x|y) is defined as,

$$H(x|y) = -\sum_{x \in X} \sum_{y \in Y} P(x,y) \log P(x|y)$$

It can be expressed as the expected value of the entropies of the conditional distributions averaged over the conditioning random variable.

Math (60 slide)

Assume that you have the joint probability of a vowel and a consonant occuring together in the same syllable

F(x, y)	p	t	k	F (y)
a	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{16}$	$\frac{1}{2}$
	$\frac{1}{16}$	$\frac{3}{16}$	0	$\frac{1}{4}$
u	0	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
F(x)	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	

$\frac{1}{16}+\frac{3}{8}+\frac{1}{16}$

$\frac{1+6+1}{16}$ 8/1

Compute the conditional probabilities:

For example

$\frac{1}{16}+\frac{1}{16}$

$$F(a|p) = \frac{F(a,p)}{F(p)} = \frac{1/16}{1/8} = \frac{1}{2}$$

$\frac{2}{\cancel{32}}$ 1/8

$$F(a|t) = \frac{F(a,t)}{F(t)} = \frac{3/8}{3/4} = \frac{1}{2}$$

Ans :-

Now compute the conditional entropy of a vowel given a constant :

$$H(V|C) = -\sum_{x \in C} \sum_{y \in V} f(x,y) \log f(y|x)$$

$$= -\Big(f(a,p) \log f(a|p) + f(a,t) \log (a|t) + \cancel{f(a,k) \log}$$

$$f(a,k) \log f(a|k) + f(i,p) \log f(i|p) + f(i,t) \log f(i|t)$$

$$+ f(i,k) \log f(i|k) + f(u,p) \log f(u|p) + f(u,t) \log$$

$$f(u|t) + f(u,k) \log f(u|k)\Big)$$

$$= -\Big(\tfrac{1}{16} \log \frac{1/16}{1/8} + \cancel{4}\, \tfrac{3}{8} \log \frac{3/8}{3/4} + \frac{1}{16} \log \frac{1/16}{1/8}$$

$$+ \tfrac{1}{16} \log \frac{1/16}{1/8} + \tfrac{3}{16} \log \frac{3/16}{3/4} + 0 + 0$$

$$+ \tfrac{3}{16} \log \frac{3/16}{3/4} + \tfrac{1}{16} \log \frac{1/16}{1/8}\Big)$$

$$= \frac{11}{8} = 1.375 \text{ bits}$$

⊛ For probability distributions we defined :-

$$F(y|x) = \frac{F(x,y)}{g(x)}$$

Conditional Entropy :-

$$H(y|x) = H(x,y) - H(x)$$

☐ Using previous theorem to the joint entropy of a consonant & a vowel is:-

$$H(C) = -\sum_{x \in C} F(x) \log F(x)$$

$$= -\big(F(P)\log F(P) + F(t)\log F(t) + F(K)\log F(K)\big)$$

~~= (1/8 + 3/~~

$$= -\left(\frac{1}{8}\log\frac{1}{8} + \frac{3}{4}\log\frac{3}{4} + \frac{1}{8}\log\frac{1}{8}\right)$$

$= 1.061$ bits.

So, the joint entropy is,

$$H(V,C) = H(V|C) + H(C)$$

$$= 1.375 + 1.061 = 2.436 \text{ bits}$$

Note:-

→ Entropy measures the amount of info in a random variable or the length of the message required to transmit the outcome.

→ Joint entropy is the number or amount of information in two (or more) random variables;

→ Conditional entropy is the amount of information in one random variable given we already know the other.

Source coding

DMS $\longrightarrow$ Binary code → Source Coding

(Discrete memoryless source

Device that perform this = Source Encoder

Objective!– To minimize the average bit rate required for a source by reducing redundancy of information source.

$$L_{min} = H(x)$$

$\bar{L}$ = Code word Length

Code efficiency, $\eta = \frac{H(x)}{\bar{L}}$

Math A scanner converts converts a black & white document line-by-line into binary data for transmission. The scanner produces sources data comprising symbols representing runs of up to six similar image pixel elements with the probabilities as shown below:-

No. of consecutive Pixels	1	2	3	4	5	6
probability of occurance	0.2	0.4	0.15	0.1	0.06	0.09

Determine the average length of a run (in pixels) & the corresponding effective information rate for this source when the scanner is traversing 1000 Pixels/s.

Ans:- Here,

Entropy, $H = \sum_{m=1}^{6} P(m) \log \left(1/P(m) \right)$

$= 0.2 \times 2.32 + 0.4 \times 1.32 + 0.15 \times 1 + 0.1\cancel{8} \times 3.32$
$+ 0.06 \times 4.059 + 0.09 \times 3.474$

$= 2.29$ bits/ symbol

Average Length is:-

$\bar{L} = \sum_{m=1}^{m} P(m)\, l_m$ (binary digits/symbol)

$= 0.2 \times 1 + 0.4 \times 2 + 0.15 \times 3 + 0.1 \times 4 + 0.06 \times 5$
$+ 0.09 \times 6$

$= 2.69$ pixels

So,

At 1000 pixels/s scan rate we

generate, $1000/2.69 = 371.747 \cong 372$ symbols/s

Thus the source info is, 2.29×372

$M = 852$ bit/s

Lecture-6

Huffman Code :-

It uses prefix's rules which assures that there is no ambiguity in the decoding process. That means no code is a prefix of another code.

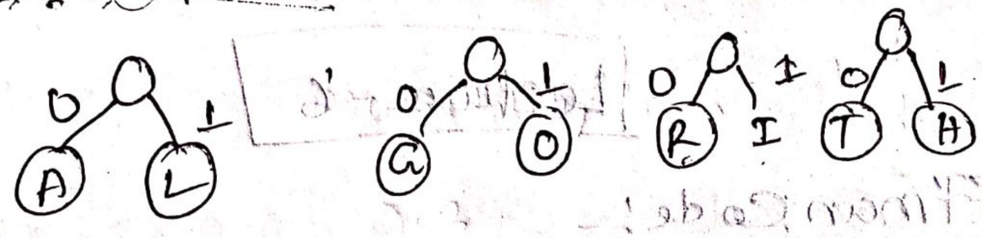
⊛ Computer এর input কে binary তে রূপান্তর করতে পারবে।

$\frac{1}{4} + \frac{2}{3}$ $\frac{7}{12}$

~~1/8, 2/4, 3/8, 4/8, 5/8, 6/8, 7/8,~~ 1. A G H I L M O R T

II

ALGORITHM

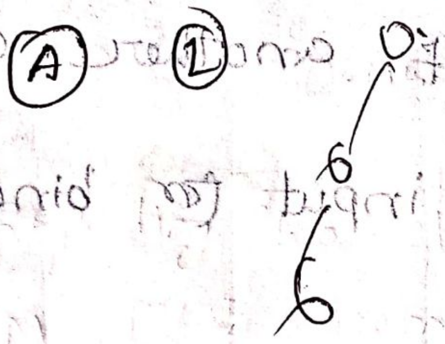


0·55 0·66 0·77 0·88 0·99

0·125 0·25 0·375 0·44 ~~0·625~~

A L G O R I T H M

~~0·875~~



Math: In a communication system the source transmits five different messages say x_1, x_2, x_3, x_4, x_5 with the prob. of 0.3, 0.15, 0.25, 0.05, 0.25 respectively. If the number of symbols used to code these messages is 2, then find the code word for each message & the coding efficiency by applying Huffman coding.

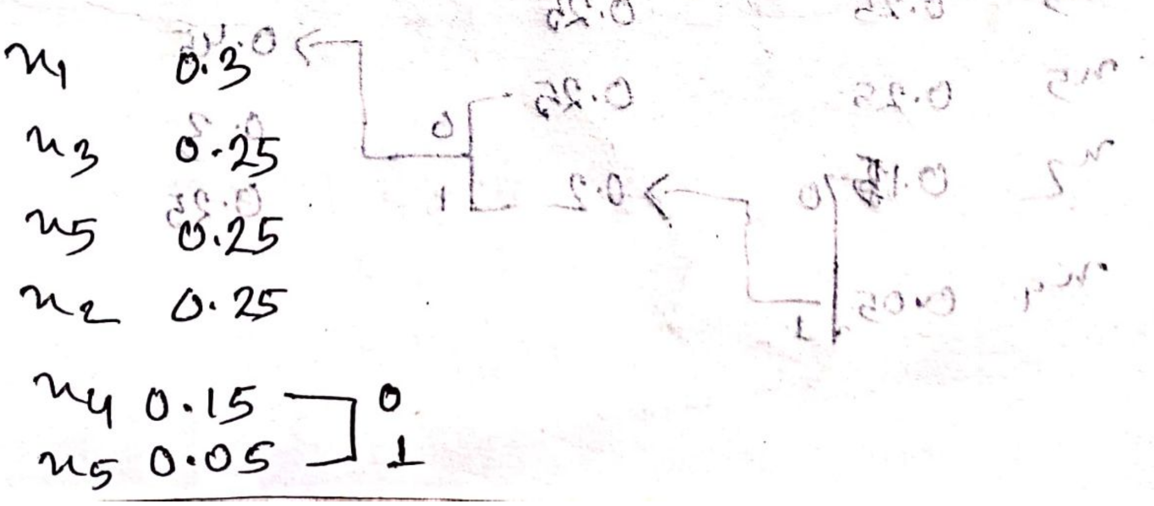
$\Rightarrow$ ~~x_4 x_2 x_3 x_5 x_1~~

~~0.05 0.15 0.25 0.25 0.3~~

$[x] = x_1 \quad x_3 \quad x_5 \quad x_2 \quad x_4$

$[P] = 0.3 \quad 0.25 \quad 0.25 \quad 0.15 \quad 0.05$

Here, $M = 2$ Let symbols assigned be 0 & 1

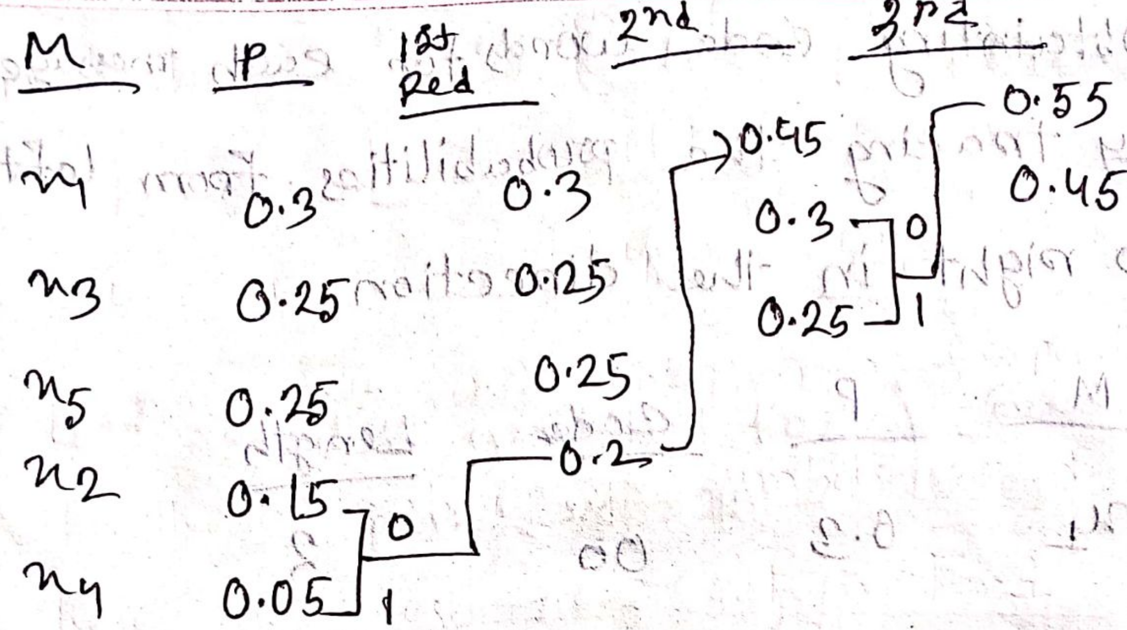


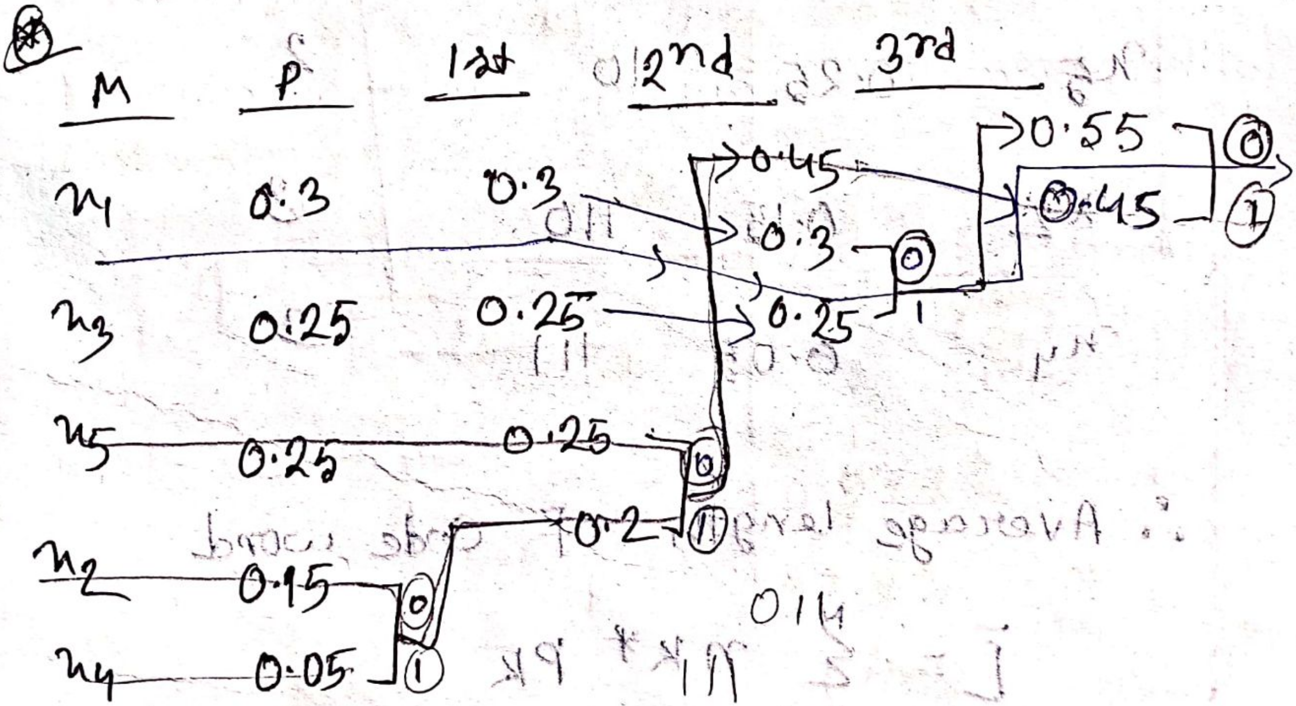
Now, adding the probability & changing its place as high as possible :-

Message	Probability		1st Reduction
x_1	0.3		0.3
x_3	0.25		0.25
x_5	0.25		0.25
x_2	0.15	0	0.2
x_4	0.05	1	

~~Repeat~~ Repeating the step 2 & 3 till only Only M prob remain :-

M	P		1st Reduction		2nd Reduction
x_1	0.3		0.3		0.45
x_3	0.25		0.25		0.3
x_5	0.25		0.25	0	0.25
x_2	0.15	0	0.2	1	
x_4	0.05	1			


M
P
1st Red
2nd
3rd
x_1
x_3
x_5
x_2
x_4
0.3
0.25
0.25
0.15
0.05
0
1
0.3
0.25
0.25
0.2
0.45
0.3
0.25
0
1
0.55
0.45


M
P
1st
2nd
3rd
x_1
x_3
x_5
x_2
x_4
0.3
0.25
0.25
0.15
0.05
0
1
0.3
0.25
0.25
0.2
0
1
0.45
0.3
0.25
0
1
0.55
0.45
0
1

Obtaining code words for each messages by tracing the probabilities from left to right in the direction

M	P	Code	Length
x_1	0.3	00	2
x_3	0.25	01	2
x_5	0.25	10	2
x_2	0.15	110	3
x_4	0.05	111	3

∴ Average length of code word

$$\bar{L} = \sum_{K=1}^{N} nK * PK$$

$$= 2 \times 0.3 + 2 \times 0.25 + 0.25 + 3 \times 0.15 + 3 \times 0.05$$

= 2.2 bi Letter/ message

$$-\{0.4\log_2 \cancel{le}(0.4) + 0.2\log_2(0.2) + (0.2)\log_2(0.2) + (0.1)\log_2(0.1) + 0.1\log_2(0.1)$$

$$= 2.12$$

Entropy,

$$H(x) = -\sum_{k=1}^{N} p_k \log p_k$$

or, $\sum p_k \log 1/p_k$ (Same)

$$= [0.3 \times \log(0.3) + 0.25\log(0.25) + 0.25\log(0.25) + 0.15\log(0.15) + 0.05\log(0.05)]$$

$$\therefore H(x) = 2.15 \text{ Bits/message}$$

~~Entropy,~~ efficiency;

$$\eta = \frac{H(x)}{\bar{L} \times \log_2 M} = \frac{2.15}{(2.2 \times \log_2 2)}$$

$$= 0.977$$

$$= 97.7\%$$

(Ans)

∴ Variance of $\bar{L}$ is,

$$\sigma^2 = \sum_{k=0}^{4} p_k (l_k - \bar{L})^2$$

$$= 0.3\times(2-2.2)^2 + 0.25\times(2-2.2)^2 + 0.25\times(2-2.2)^2 + 0.15\times(3-2.2)^2 + 0.05(3-2.2)^2 =$$

$= 0.012 + 0.01 + 0.01 + 0.06 + 0.02$

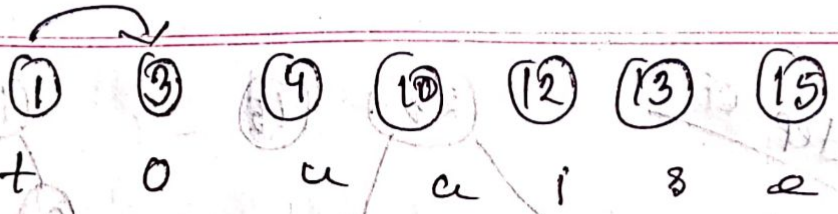
$= 0.084$

(Ans)

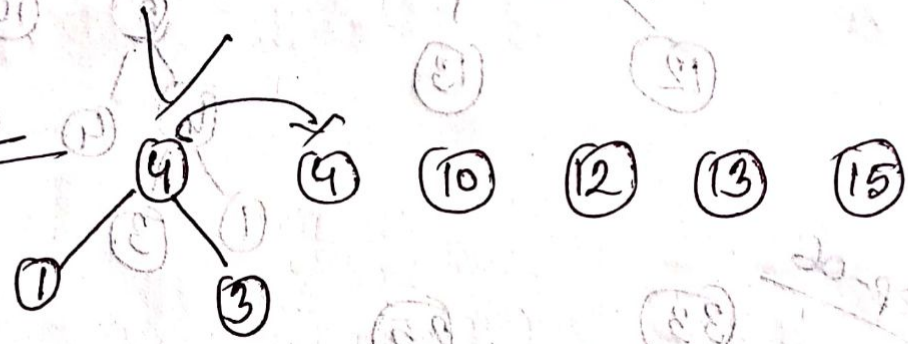
problem (~~with~~ Huffmen with the help of Frequency)

Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1

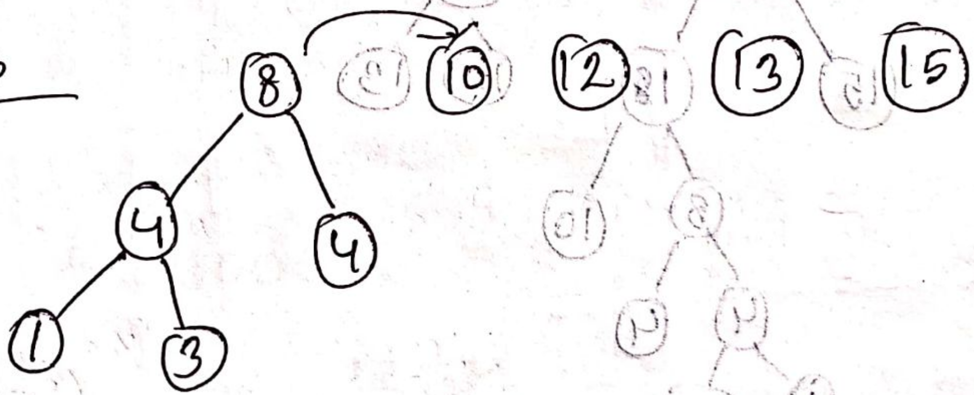
Step 01



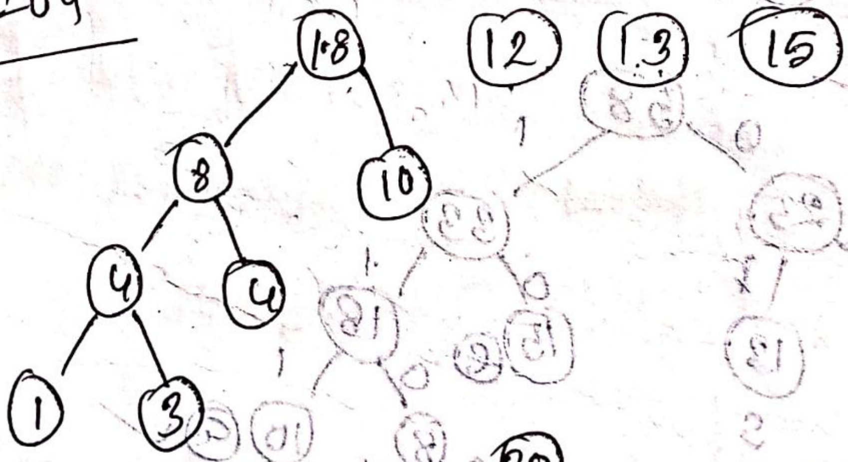
Step-02



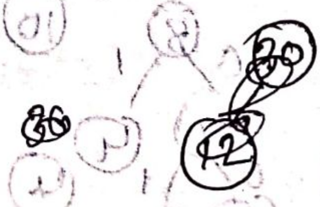
Step-03



Step-04

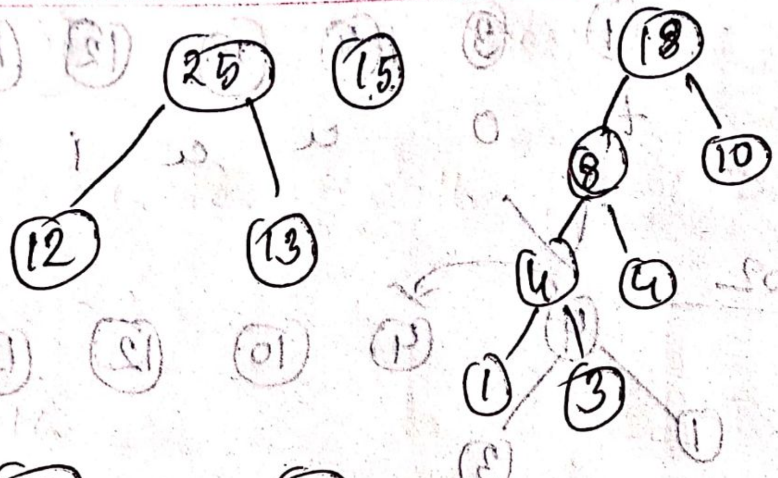


~~Step-05~~

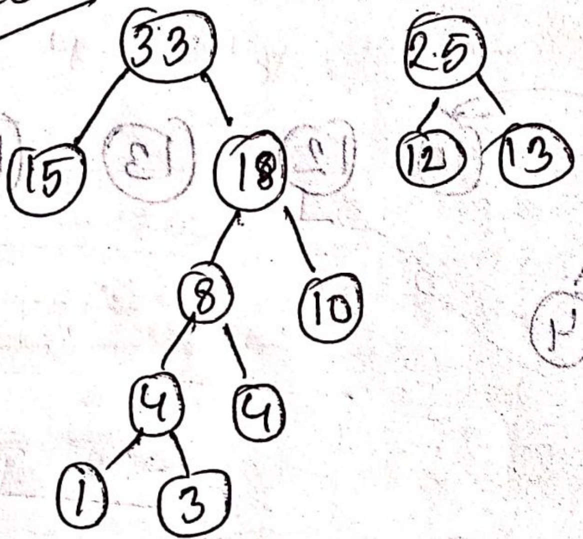


P.T.O

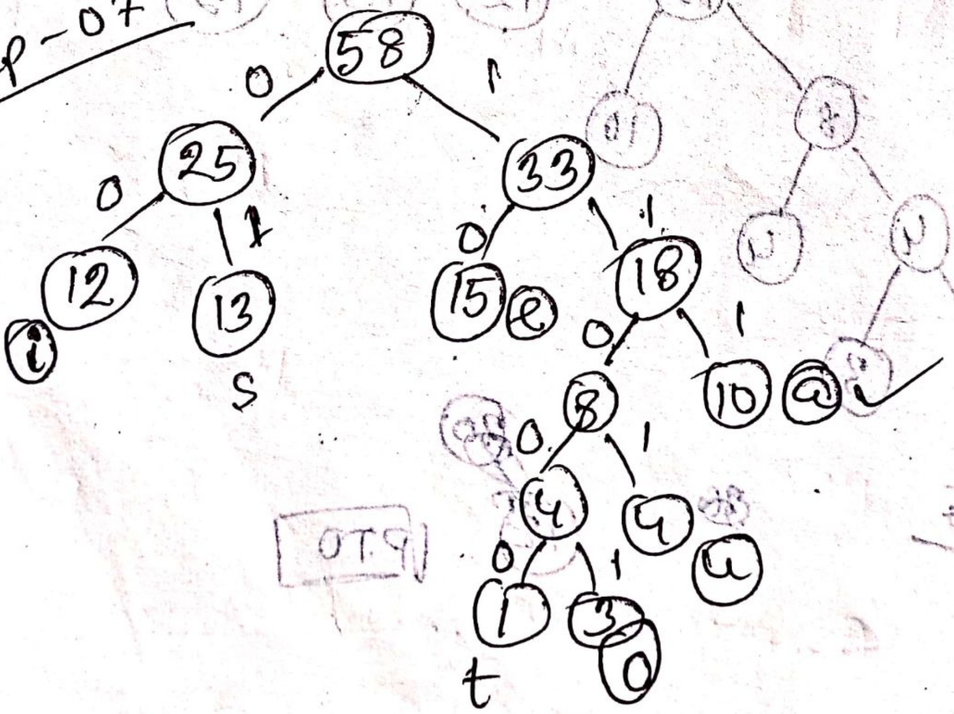
Step-05



Step-06



Step-07



Converting them using Huffman code :-

a = 111

e = 10

i = 00

o = 11001

u = 1101

s = 01

t = 11000

Average code length :-

~~$\bar{L} = \Sigma$~~

~~Av~~ Average code length

$= \Sigma(\text{Frequency} \times \text{code length}) / \Sigma(\text{Frequency})$

$= \{(10\times3) + (15\times2) + (12\times2) + (3\times5) + (4\times4) + (13\times2)$
$(1\times5)\} / (10+15+12+3+4+13+1)$

$= 2.52$

Length of encoded message :-

Average code length per character

= 58 × 2.52

= 146.16

= 147 bits

Lecture-07

prefix code :-

Suppose :-

A = 00

B = 010

C = 011

D = 10

E = 11

Here, none of the Binary match with others. ~~Moreover~~, Means, A = 00 & there is no other code which starts with 00. Same happens to others.

But if we change it for a moment:-

A = 00
B = 0101
C = 001 → 011 to 001
D = 10
E = 11

Here the system is still one to one correspondence. However, it is no longer a prefix-free code system. Cause there is 00 is A & C.

LEMPEL-ZIV CODING

Data Compression = Compaction

The process of reducing the amount of data needed for the storage or transmission of a given piece of information by the use of encoding techniques.

Data Compression:- ① Loseless (Exact)
② Lossy (Inexact)

Loseless:- Can be reversed to original.

Necessary for text (Every character is important)

Lossy:- Loses details. Have small errors.

~~Can't b~~ Can be accepted in image or voice

There are ~~the~~ three most used compression

① Zip (windows)

② StuffIt (Apple computers)

③ gzip (Unix)

Lample-Ziv Algorithm:-

① Invented by Lempel, Ziv & Terry welch

→ Addresses spatial redundancies in an image

→ It is an error free compression approach

→ prior knowledge of probability of occurrence of symbols to be encoded is not required.

LZW-Coding:-

Most palette color images LZW yields the high compression efficiency without sacrificing image data.

Most GIF use the LZW compression.

LEMPEL-ZIV CODING Example

000101110010100101 - - -

Codebook Index	1	2	3	4	5	6	7	8	9
Subsequence	0	1	00	01	011	10	010	100	101
Representation			11	12	42	21	41	61	62
Encoding			0010	0011	1001	0100	1000	1100	1101

ও [illegible]:

→ Subsequence এর

Example-02

ABAA BABA BBA BABABA BB ABB AAB

Position	1	2	3	4	5	6	7	8	9
Sequence	A	B	AA	BA	BAB	BABA	~~BABAB~~ BABAB	BABB	AAB
Representation	ϕA	ϕB	1A	2A	4B	5A	6B	5B	~~4A~~ 3B
Encoding	0	1	10	100	1001	1010	1101	1011	111
	0000	0001	0010	0100	1001	1010	1101	1011	0111

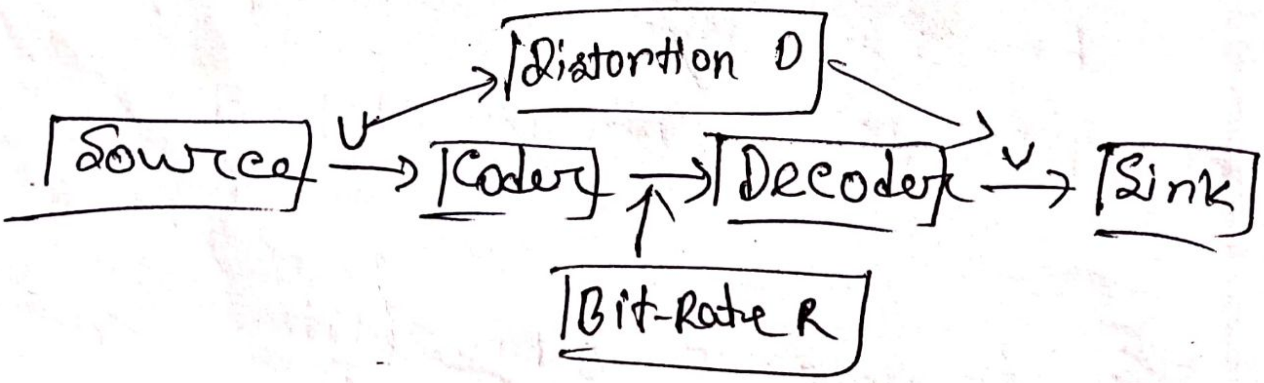
Rate Distortion Theory:-

Theoretical Foundation for lossy data compressio

Addresses:- The minimal number OF bit/symbol as measured by the rate R. That should be Communicated over a ~~signal~~ channel.

Input signal can be approximately reconstructed at the receiver (output sig.) without exceeding an expected distortion D.

- ⊛ It calculates minimum ~~number~~ transmission bit-rate R for a required picture quality
- ⊛ Results are obtained without consideration of a specific coding method.



Symbol $u \rightarrow$ sender

$v \rightarrow$ Receiver

Per-symbol distortion:-

$$d(u,v) \geq 0$$

$$d(u,v) = 0 \quad \text{For } u = v$$

Average Distortion:-

$$D(u,v) = E\{d(u,v)\} = \sum_{u} \sum_{v} P(u,v) \cdot d(u,v)$$

Distortion Criterion:-

$$D \leq D^* = d_{Max}$$

→ Maximum Average Distortion

MPEG Audio Compression

MPEG = Moving Picture Experts GROUP

It is a standard for both ~~vido~~ video & audio. Compression. From noisy signals which is transmitted from Satellite.

It is first international algorithm for digital audio compression that showed high fidelity.

Data Compression is a technique in which data content of the input signal to system is compressed so that original signal is filtered out & unwanted or undesired signals are removed and true digital signal is obtained as output.

High quality audio transmission & storage are the basic goals of audio compression

So, MPEG compresses audio signal to make it audible to the human ear & remove the unwanted signals introduces in the communication path without assuming nature of audio source.

Audio Types

Level-1 :- Low Complexicity

Level-2 :- Musicam (DVB)

Level-3 :- Mp3

Advanced audio coding.

MPEG Layer-1 :-

- This process divides the digital audio signal into multiple frequency bands & only transmits the audio bands that can be heard by the listener.

MPEG Layer-2 :-

It is known as Musicam system.
It achieves medium compression ratios dividing the audio signal into sub bands, coding these sub bands & multiplexing them together. It is used in DAB.

MPEG Layer-3 :-

Lossy audio coding standard : The MP3 system achieves high-compression ratios (10:1) by removing redundant info & sounds that the human ear can't detect or perceive.

The signal which can't be detected is called as Psychoacoustic compression.
64 Kbps per audio channel is needed to obtain high fidelity quality.

~~ve~~ Video Compression

Video takes up so much space ~~17MPps~~ Uncompressed footage from a camcorder takes up about 17MBps.

2 kinds of compression:-

① Lossy video compression

② Loseless video compression

1 Algorithm of MPEG compress data to form small bits that can be easily transmitted & then decompressed.

It achieves high compression rate by storing only the changes from one frame to another. ~~So then~~

The video in Encoded using Discrete Cosine transform (DCT). MPEG uses a type of lossy compression, since some data is remov

JPEG :-

Joint Photographic Experts Group.

It is a lossy image compression method.

It uses DCT method.

It allows a tradeoff between storage size & the degree of compression can be adjusted.

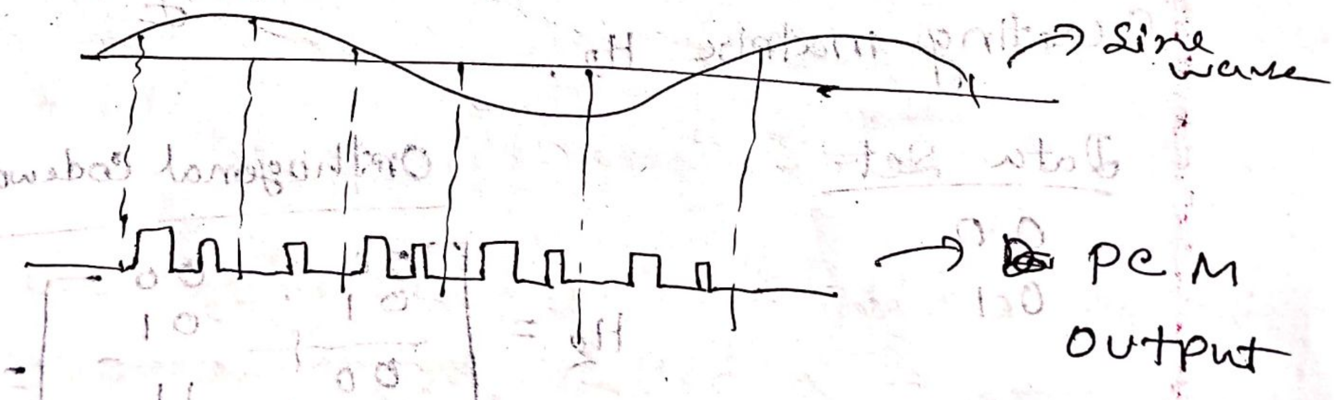
As JPEG discards much of the original info, as a result image can't be reconstructed from a JPEG file, It is lossy compression.

A photograph contains considerable information that the human eye can't detect so this can be safely discarded.

Waveform coding

It is the type of speech digitization used in the public switched telephony network.

⇒ world's communication systems have converted to a digital transmission format called pulse code modulation (PCM) → It is coded form of the original voice



~~Ot~~ Orthogonal Codes

A one-bit data set can be transformed, using ~~ot~~ orthogonal code words of two digits each. Described by the rows of matrix H_1 as follows:

Data Set **Orthogonal Codeword Set**

$$\begin{array}{c} 0 \\ 1 \end{array} \qquad H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

To encode a 2bit data set, we extend the foregoing set both horizontally & vertically, creating matrix H_2.

Data Set **Orthogonal Codeword Set**

$$\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \qquad H_2 = \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right] = \begin{bmatrix} H_1 & H_1 \\ H_1 & \overline{H_1} \end{bmatrix}$$

The lower right quadrant is the complement of the prior codeword set. We continue the same construction rule to obtain an orthogonal set H_3 for a 3-bit data set.

Data Set L

000
001
010
011

100
101
110
111

$$H_3 = \left[\begin{array}{c|c} 0000 & 0000 \\ 0101 & 0101 \\ 0011 & 0011 \\ 01\cancel{0}10 & 0110 \\ \hline 0000 & 100 \\ 0101 & 101 \\ 0011 & 110 \\ 0110 & 111 \end{array}\right]$$

$$H_3 = \begin{bmatrix} H_2 & H_2 \\ H_2 & \overline{H_2} \end{bmatrix}$$

⊛ We can construct a codeword set H_K, of dimension $2^K \times 2^K$, called a Hadamard matrix, for a K-bit data set from the H_{K-1}

$$\therefore H_K = \begin{bmatrix} H_{K-1} & H_{K-1} \\ H_{K-1} & \overline{H_{K-1}} \end{bmatrix}$$

$\log_2 2$

$2 \rightarrow$ Base

$\log_{10} 10$

$\boxed{10} \rightarrow$ Base

(1) (b) Entropy of source

Entropy, $H = -\sum P_i \log_2 P_i$

$= -0.30 \log_2(0.30) - 0.10 \log_2(\cancel{0.03}\ 0.10)$

$- 0.02 \log_2(0.02) - 0.15 \log_2(0.15) - (0.40)\log_2(0.4)$

$- 0.03 \log_2(0.03)$

$= 0.521 + 0.3322 + 0.113 + 0.411 + 0.529$

$+ 0.152$

$= 2.05724$

$= \cancel{2.0582}$ bits / symbol

∴ Information Rate, $R = H \cdot R_s$

$= 2.0582 \times 9.6 \times 1000$

$= \cancel{19.758}\ 19749.504$

[bits/s]

(ii) Data rate = 3×9600 (bits/s)

$= 28,800$ [bits/s]

2(a)

~~$H(x) = -\sum P_k \log_2 P_k$~~

~~$= -\frac{1}{20} \log_2 \frac{1}{20} +$~~

$\frac{1}{20} + 0$

F(x)

x \ y	0	1	F(y)
0	1/20	1/20	1/10
1	0	1/20	1/20
F(x)	1/20	1/10	

$\frac{1}{20} + \frac{1}{20}$

$\frac{2}{20}$

$H(x) = -\sum F(x) \log(F(x))$

$= -\left(\frac{1}{20} \log_2 \frac{1}{20} + \frac{1}{16} \log \frac{1}{10}\right)$

$= 0.5483$ bits~~/symbol~~

$H(y) = -\sum P(y) \log P(y)$

$= -\left(\frac{1}{10} \log \frac{1}{10} + \frac{1}{20} \log \frac{1}{20}\right)$

$= 0.5483$ bits

(ii) $H(x|y)$, $H(y|x)$

$H(x|y) = -\underset{x\in X}{\sum}\underset{y\in Y}{\sum} P(x,y) \log P(y|x)$

$= -P(0,0) \log P(0|0) - P(0,1) \log P(0|1)$

$\quad - P(1,0) \log P(1|0) - P(1,1) \log P(1|1)$

$P(0|0) = \frac{P(0,0)}{P(x)} = \frac{1/20}{1/20} = 1$

$P(0|1) = \frac{P(0,1)}{P(x)} = \frac{1/20}{1/10} = \frac{1}{2}$

$P(1|0) = \frac{P(1,0)}{P(x)} = \frac{1/20}{0} = \frac{0}{1/20} = 0$

$P(1|1) = \frac{P(1,1)}{P(x)} = \frac{1/20}{1/10} = \frac{1}{2}$

$$= -\Big\{ \tfrac{1}{20} \log_2(1) + \tfrac{1}{20} \log_2 (\tfrac{1}{2})$$

$$+ 0 \log_2 (0) + \tfrac{1}{20} \log_2 (\tfrac{1}{2}) \Big\}$$

$$= \Big(= 0 - \tfrac{1}{20} + 0 - \tfrac{1}{20} \Big)$$

$$= - \left(- \frac{1+1}{20} \right)$$

$$= \frac{2}{20} = \tfrac{1}{10} \text{ bits}$$

$$H(y|x) = - F(0,0) \log F(0|0)$$

$$- F(0,1) \log F(0|1) ~~- F(1,0) \log F(1|~~$$

$$- F(1,0) \log F(1|0) - F(1,1) \log F(1|1)$$

$$= -\Big\{ \tfrac{1}{20} \log_2(1) + (\tfrac{1}{20}) \log_2(\tfrac{1}{2}) + 0 \log_2(0)$$

$$+ \tfrac{1}{20} \log_2 (\tfrac{1}{2}) \Big\}$$

$$= \tfrac{1}{10} \text{ bits}$$

(iii) $H(x,y)$

$$= -\sum_{x\in X}\sum_{y\in Y} F(x,y)\log F(x,y)$$

$$= -\left\{\frac{1}{16}\log_2\frac{1}{16} + 0\log_2 0 + \frac{1}{20}\log_2\frac{1}{20} + \frac{1}{20}\log_2\frac{1}{20}\right\}$$

$= 0.6822$ bits

(iv) $H(y) = -\sum F(y)\log_2 F(y)$

$= 0.5483$ bits

$H(y|x)$ ~~$= -\sum_{x\in X}\sum_{y\in Y} F(x,y)\log_2 F(x|y)$~~

$$= -\sum_{x\in X}\sum_{y\in Y} F(x,y)\log_2(\cancel{x}y|x)$$

$= \frac{1}{10}$ bits

$H(y) - H(y|x) = \left(\overset{0.5483}{\cancel{0.6822}} - \frac{1}{10}\right)$ bits

$=$ ~~0.5822 bits~~ 0.4483 bits/symbol

Mutual Information (v) $I(x;y) = H(x) - H(x|y)$

$= 0.5483 - \frac{1}{10}$

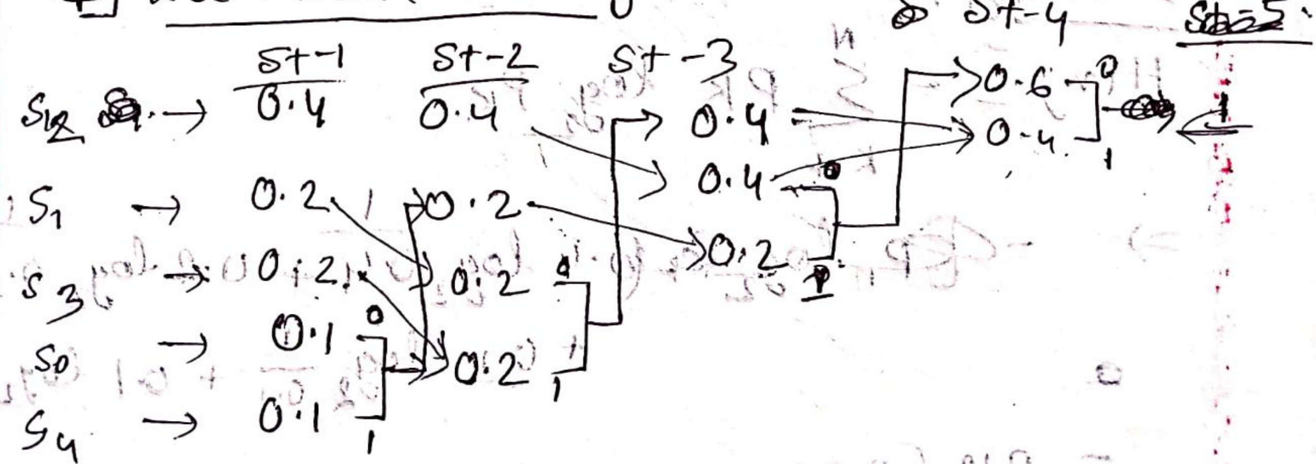
$= 0.4483$ bits/symbol

4 2 1
1 0 1

2 (b) ABAABABA BBABA BABA BBABBA ABBA BABAABAA

position	1	2	3	4	5	6	7	8	9	10	11	12
Sequence	A	B	AA	BA	BAB	BABA	BABAB	BABB	AAB	BABABA	AB	AA
Representation	ϕ A	ϕB	1A	2A	4B	5A	6B	5B	3B	7A	1B	3
Encoding	0	1	10	100	~~1011~~ 1001	1010	1101	1011	0111	1110	11	011
	0000	0001	0010	0100	1001 ~~1011~~	1010	1101	1011	0111	1110	0011	0011

Huffman Coding :-



code

$S_2 \rightarrow 00 \rightarrow (2)$
$S_1 \rightarrow 10 \rightarrow (2)$
$S_3 \rightarrow 11 \rightarrow (2)$
$S_0 \rightarrow 010 \rightarrow (3)$

$S_4 \rightarrow 011 \rightarrow 3$

Average code word length :-

$$\bar{L} = \sum_{K=1}^{K=1} NK * PK$$

$$\bar{L} = n_1 * P_1 + n_2 * P_2 + n_3 * P_3 + n_4 * P_4 + n_5 * P_5$$

$$= 2\times 0.4 + 2 \times 0.2 + 2\times 0.2 + 3\times 0.1 + 3\times 0.1$$

$$= 0.8 + 0.4 + 0.4 + 0.3 + 0.3$$

$= 2.2$ letters/messages

Entropy :-

$$H(x) = -\sum_{K=1}^{N} P_K \log_2 P_K$$

$$\Rightarrow -~~\Sigma P_1 \log_2 P_K~~ \left(0.4 \log_2 \frac{1}{0.4} + 0.2 \log_2 \frac{1}{0.2} + 0.1 \log_2 \frac{1}{0.1} + 0.1 \log_2 \frac{1}{0.1}\right)$$

$= 2.12$ (Ans)

(iii) The variance

$$\sigma^2 = \sum_{K=0}^{4} P_K (n - \bar{L})^2$$

$= \cancel{(2-2.2)+(2-2.2)^2+(2-2.2)^2}$

$= 0.4\,(2-2.2)^2 + 0.2\,(2-2.2)^2 + 0.1\,(2-2.2)^2 + 0.1\,(2-2.2)^2$

$= 0.16$ (Ans)

THE END ☺

KEEP
CALM
AND
GET READY FOR
FINAL EXAM!

MD IFTAKHAR KABIR SAKUR

25th BATCH

COMPUTER AND COMMUNICATION ENGINEERING

International Islamic University Chittagong

COURSE CODE: CCE-3503

COURSE TITLE: Information Theory and Error Coding

COURSE TEACHER:

PROF. RAZU AHMED

&

Md Humayun Kabir

Part – A: 20-Marks

1. **Channel Capacity and Coding:** Discrete channels, mutual information, Properties of Mutual Information, channel capacity, Shannon's channel coding theorem, bandwidth S/N trade-off. Channel capacity theorem.
2. **Model of digital communication system employing coding.** History of Coding, Types of Coding, Types of Decoding, Types of Error Control, Code Rate and Redundancy, Hamming distance, Hamming weight, Hamming bound. parity check codes, Hamming codes, Cyclic Codes.

Part – B: 30-Marks

1. Linear block codes, generator and parity check matrix, syndrome decoding. Cyclic codes, generation and detection. Coding for reliable communication, coding gain, bandwidth expansion ratio. Comparison of coded and uncoded systems.
2. Convolutional Codes, Encoding, Encoder representation, impulse Response of the Encoder, polynomial Representation, state representation and the State Diagram, Code tree, Trellis diagram, Turbo coding, LDPC
3. Decoding of convolutional codes, Veterbi's algorithm, sequential decoding. Transfer function and distance properties of convolutional codes. Error Correction Capability of Convolutional Codes, Coding gain.

Error Detection

⊞ Vector space :-

The set of all binary n-tuples v_n is called a vector space over the binary field of two element (0 & 1)

Binary field → Addition (X-OR)

Binary field → Multiplication

Addition (X-OR)	Multiplication
$0 \oplus 0 = 0$	$0 \cdot 0 = 0$
$0 \oplus 1 = 1$	$0 \cdot 1 = 0$
$1 \oplus 0 = 1$	$1 \cdot 0 = 0$
$1 \oplus 1 = 0$	$1 \cdot 1 = 1$

↳ (Modulo-2 operation)

⊛ Errors are detecting using parity check:-

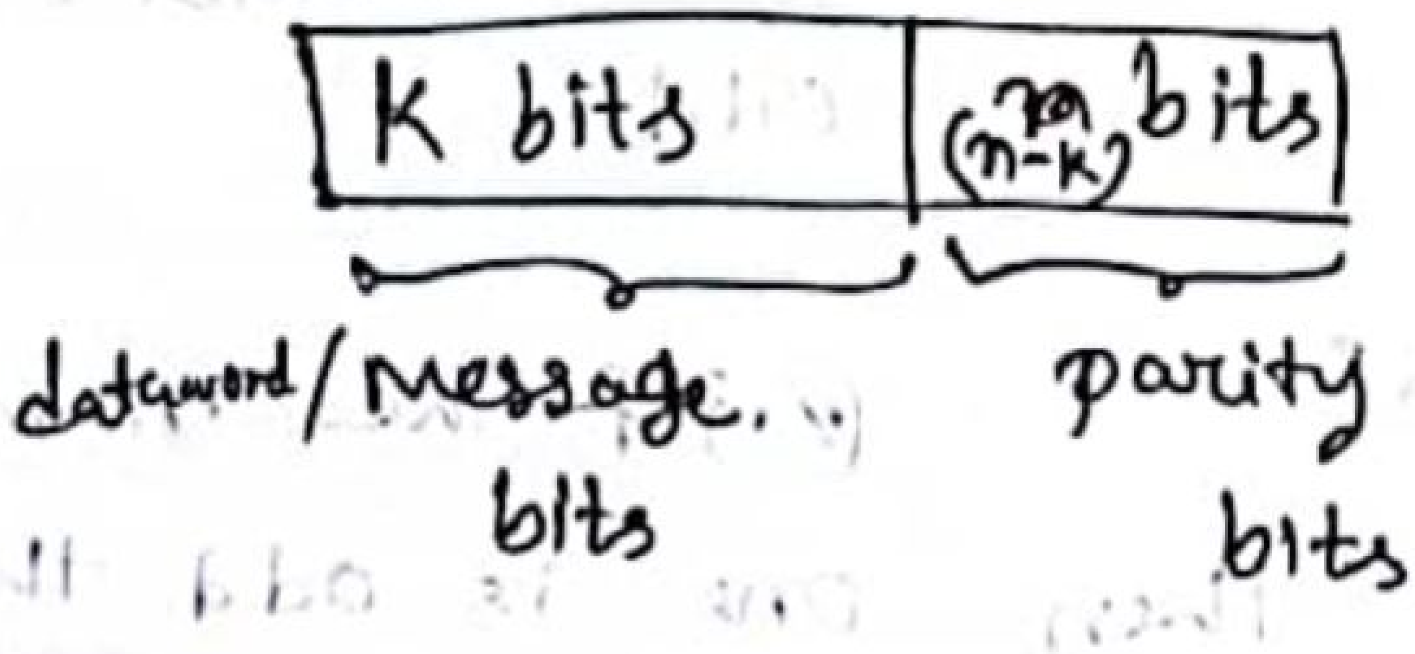
→ even parity	Odd parity
① The number of bits with a value of one are ~~connected~~ counted. If the value of 1 is odd we ~~make~~ add another 1 to make it even.	① If the number of bits with a value of one is an even number, the parity bit value is set to one to make the entire number of 1 as odd.
② If the value of even is even then we set the value of parity bit as 0.	② If the number of one is odd then we add zero to keep it odd.

~~Error Detection~~

~~Vector Space~~

Linear Block Coding :-

Message bits are divided into two blocks. Each block contains K bits, and each K bits of a block defines a dataword.



The possible Codewords will be 2^n out of which 2^k contains datawords.

During transmission, if errors are introduced then this codewords will be changed.

In terms of Codewords and datawords, a term code rate is used,

$$r_c = \frac{k}{n}$$

Code rate:- Ratio of dataword bits to codeword bits.

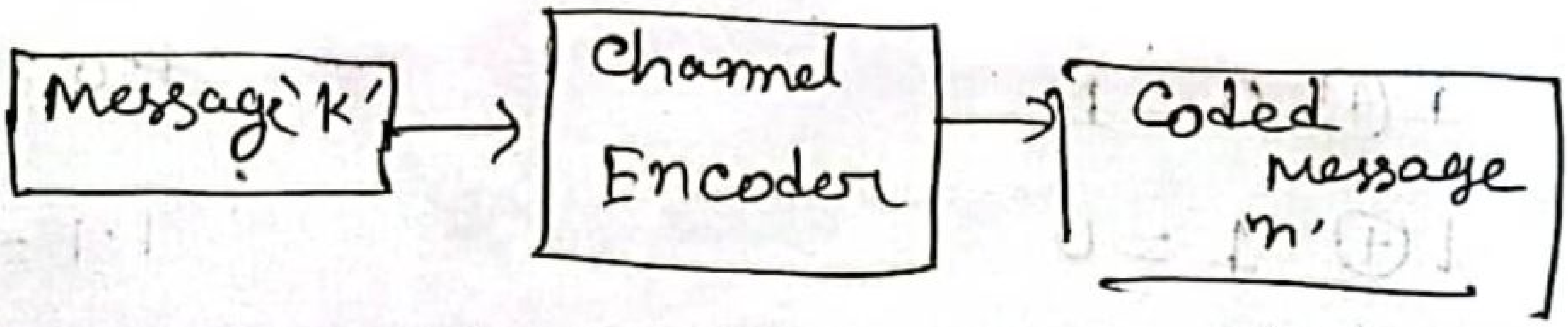
Linear Block codes the data information is divided into ~~two~~ blocks of length k-bits data word.

$n = k + r$ → parity bits.

Vector notation is used for the Data word and code word

→ For Data word, $m = (m_1, m_2, \ldots, m_k)$

→ For Codeword, $u = (u_1, u_2, \ldots, u_n)$



There is always one unique codeword for each data word.

$$\text{Code rate} = k/n$$

Q

Generator Matrix

$$\begin{array}{r}011010\\ \oplus\ 101001\\ \hline 110011\end{array}$$

$$G = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1\,1\,0 & 1\,0\,0 \\ 0\,1\,1 & 0\,1\,0 \\ 1\,0\,1 & 0\,0\,1 \end{bmatrix}$$

v_1, v_2 & v_3 are three linearly independent vectors (a subset of 8 code vectors) that can generate all the code vectors.

(এগুলো এমন codeword হবে সেগুলো বাকি সব code vectors generate করতে পারবে)

Formula of Code Generate

$$U = m \times G$$

m = Message vector

U = Codeword

For ex! -

$$u_4 = [1\,1\,0] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= 1 \cdot v_1 + 1 \cdot v_2 + 0 \cdot v_3 = v_1 + v_2$$

$\overset{n}{(6},\overset{k}{3)}$ 6 3

2^n = Tuples [illegible] 2^{n} = Tuples.

2^k = vector / [illegible] dimension [illegible] or

Systematic Linear Block code

A systematic (n,k) linear block code is a mapping from a k-dimensional message vector to an n-dimensional codeword in such a way that part of the sequence generated coincide with the k message digits. The remaining (n,k) digits are parity digits.

$$G = [P : I_k]$$

Given, message k-tuples

$$m = m_1, m_2, m_3, ---, m_k$$

general code vector n-tuples,

$$V = u_1, u_2 --- u_n$$

VVIm

Patni

$n \to$ code word $\to$ message

$k \to$ Tuples

2^k = message generated $= 2^3 = 8$

Question-

For a $\overset{n\ k}{(6,3)}$ code, the generator matrix

G is,

$$G = \left[\begin{array}{cccccc} \underbrace{\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}}_{I} & \underbrace{\begin{matrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{matrix}}_{P}\end{array}\right]$$

Find,

(a) All corresponding code vectors

(b) Minimum Humming Distance

(c) verify that this code is a single-error correcting code.

(d) parity check Matrix.

(e) Determine Transmitted codeword if received codeword is : 100011

(কয়টা one আছে সেই সংখ্যাটাই Humming distance)

∴ Minimum Humming distance

$d_{min} = 3$

(Ans)

ⓒ

This code is single bit error coding, Correcting code,

① $d_{min} \geq s+1$ [Here, s = No. of error detection]

$\therefore 3 \geq s+1$

$\Rightarrow 2 \geq s$

or $s \leq 2$

That means, this code detect 2 bit error in 6 bit message.

From error pattern

we get,

Syndrome **Error pattern**

100 100000

সুতরাং, $r = u + e$

$u = r + e$

$= 001110 + 100000$

$= 101110$

Class Test Question

[Q] Consider a (5,2) linear Systematic block code defined by the generator matrix,

$$G = \begin{bmatrix} g_0 \\ g_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

P

(a) Find all the code ~~words~~ vector of the above block code.

(b) Find the parity check Matrix for this code

(c) How many errors can the code correct

(d) Find all the code words of the above block code

(Ans)

(a)

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

I P

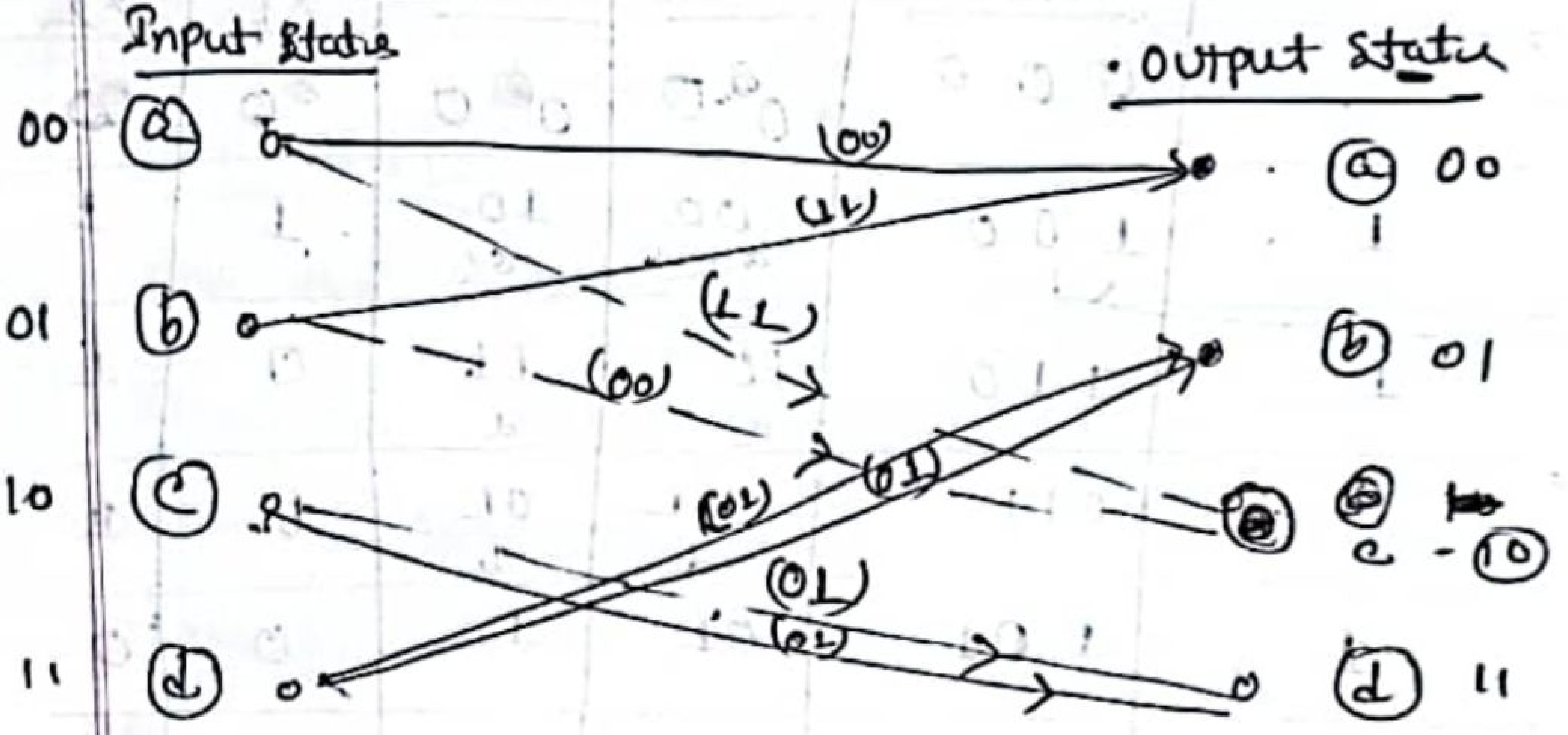
Here,

The linear system is $(5,\overset{k}{2})$

Here,

message bit = 2 ~~bit~~ tuples

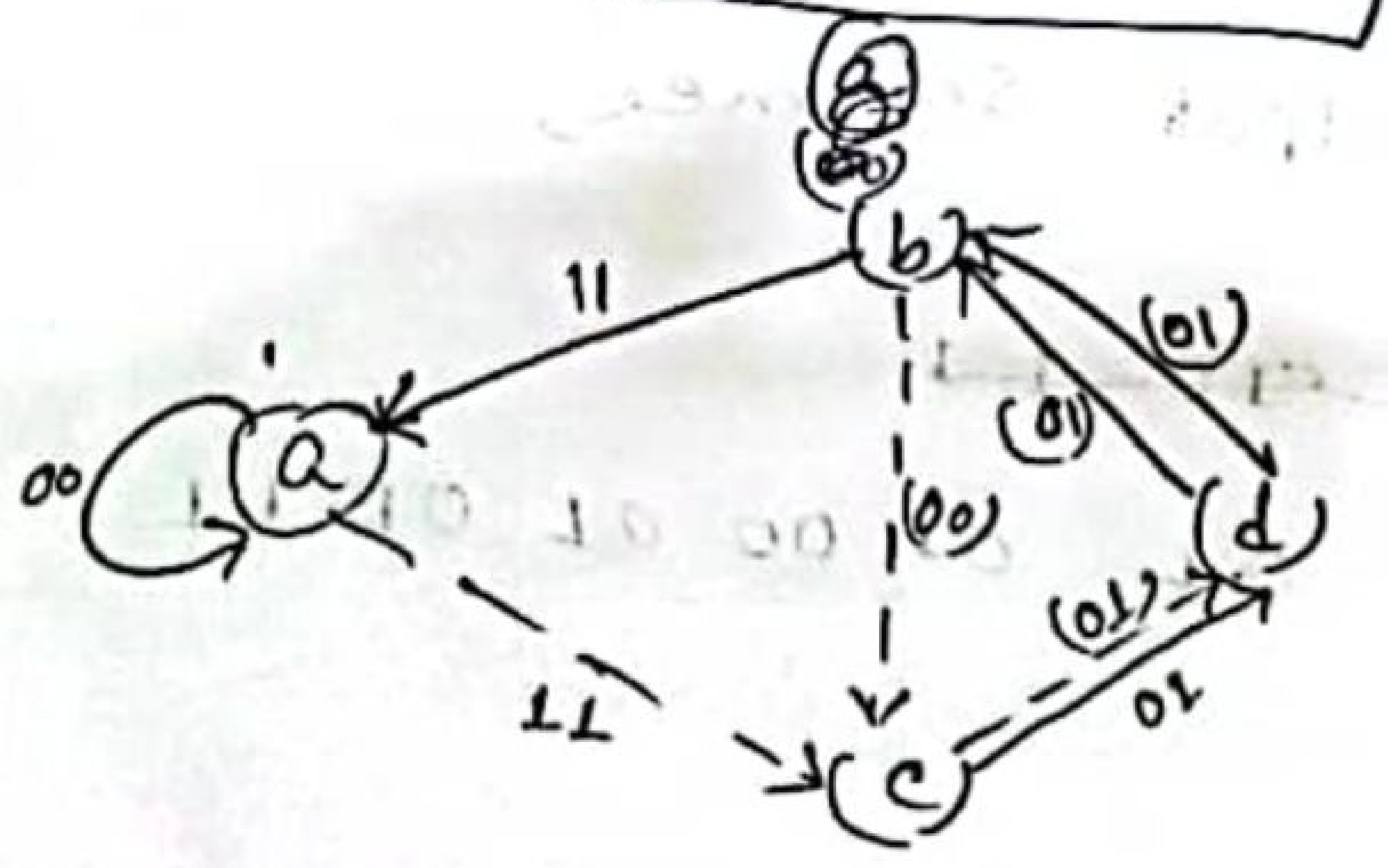
Code Trellis diagram



0→ ——— (Solid line)

1→ - - - - - (dot line)

State Diagram



O/P sequence এ স্থান পাবে না

এটা current state

Input	Register contents	Current state State at time t_i	Next state State at time t_{i+1}	Branch word at time t_i	
				x_1	α_2
–	000	00	00	0	0
1	100	00 a (a)	10 c	1	1
1	110	10 c	11 d	0	1
0	011	11 d	01 b	0	1
1	101	01	10	0	0
1	110	10	11	0	1
0	011	11	01	0	1
0	001	01	00	1	1

So the Output Sequence,

~~U= 11 010 11~~

U = 11 01 01 00 01 01 11

Another type of error correcting code where the output bits are

---

$p = n - k$

Code rate, $r = \frac{k}{n}$

k = no. of $\overset{msg}{\cancel{msg}}$ bit;

n = no. of encode o/p bits

K = Constraint length
↓
single message bit influences encoder o/p for different successive shift.

Shifting alway

$2^3 = 8$ bit হতে 2 কমে

$8-2=6$ বার shifting হবে।

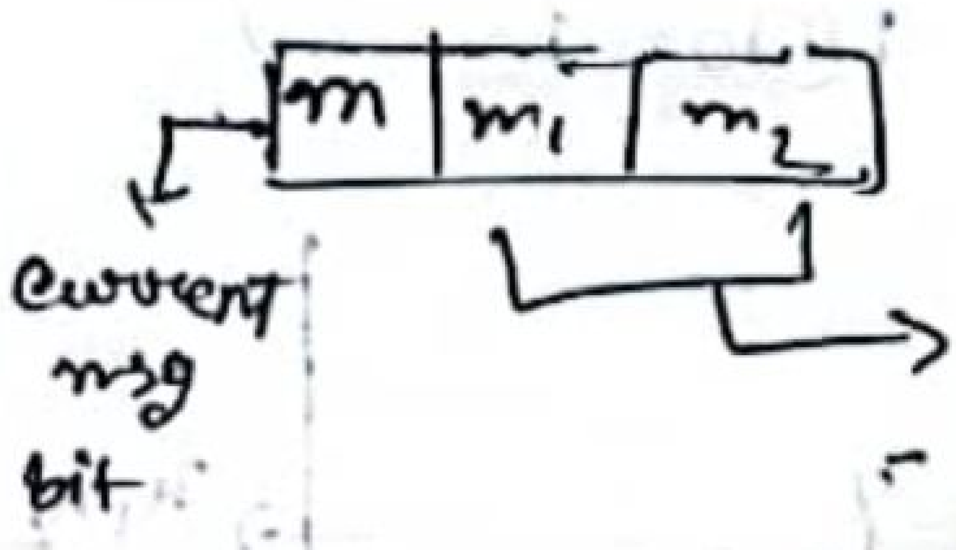
$K=3$,

$K-1=3-1=2$ টি zero দিয়ে register কে Flash দিবে

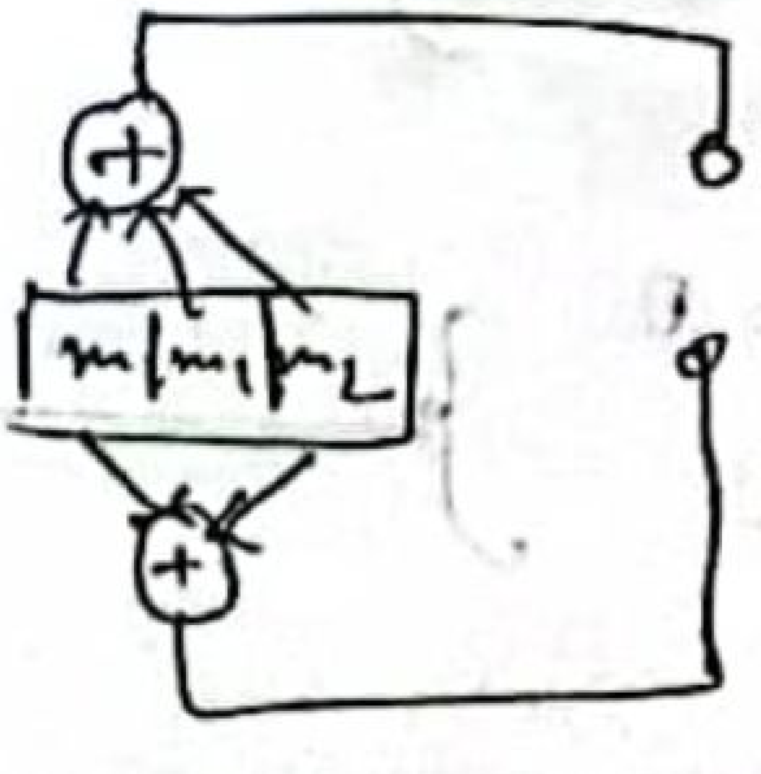
Code Dimensions: $(n,k) = (2,1)$

Convolution Code States
&
Code tree

Initially all registers are 0.



→ State of convolution codes



$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

m_1 & m_2
→ Define the states

1→0→0   1→1→0

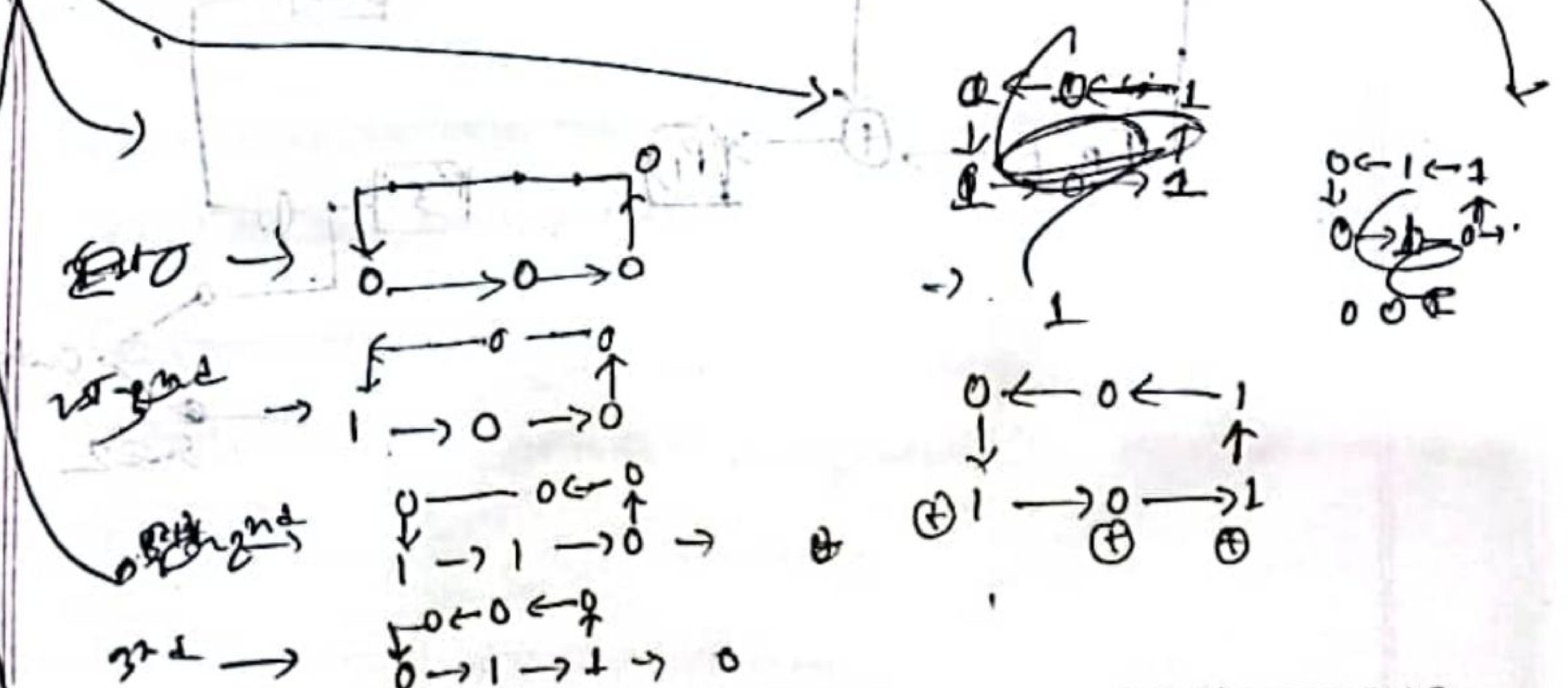
Input queue	Shifting number	Register content	Output
1011	0	000	—
101	1	110	1
10	2	101	1
1	3	101	0
—	4	100	1

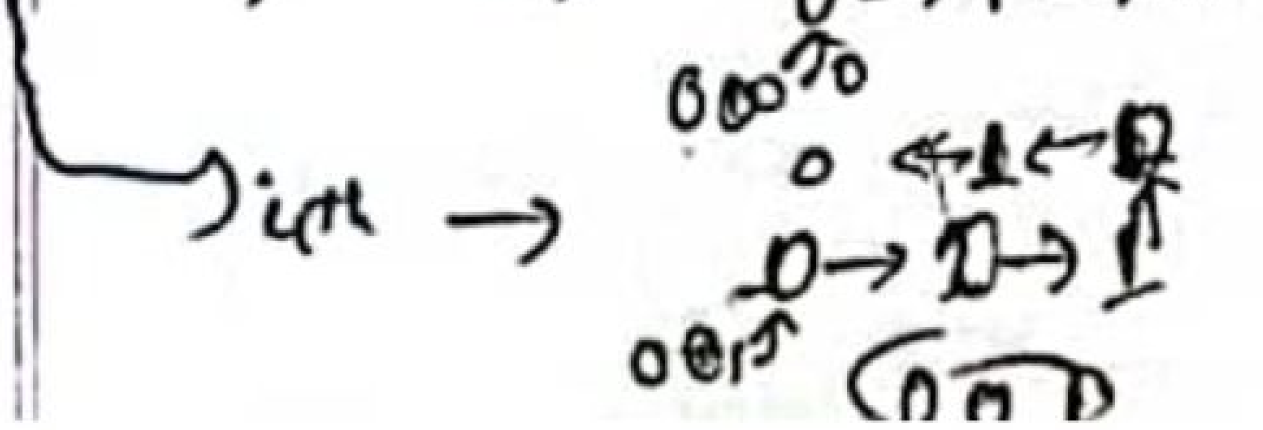
$$\text{Code word} = p(x) + (x)^{n-k} \cdot m(x)$$

$$= 1001011$$

- error pattern table.
- Location error pattern,
- Convolution Coding
- convolution encoder designation
- Impulse Response encoder
- Encoder State Diagram
- Trellis diagram
- Mutual Information [Mid PLFC]

Input queue	Shift number	Register content	Output & Feedback
0001011	0	000	–
000101	1	100	0
00010	2	110	0
0001	3	011	0
000	4	~~011~~ 001	1
00	5	111	1
0	6	101	1
–	7	100	1





∴ codeword = ~~1001111000~~
= $P(x) + x^{n-k} m(x)$
= 1001111000

$$= \frac{x^3 + x^5 + x^6}{x^3 + x + 1}$$

so,

~~$x^3+x+1 \,)\, x^3 + x^5 + x^6 \,(\, 1 + x^2 + x^3$~~

~~$x^3 + x^5 + 1$~~

~~$x^5 + x^6 + x + 1$~~

~~$x^5 + x^3 + x^2$~~

~~$x^6 + x + 1 + x^3 + x^2$~~

~~$x^6 + x^4 + x^3$~~

~~$x + 1 + x^2 + x^4$~~

$x^3+x+1 \,)\, x^3 + x^5 + x^6 \,(\, 1 + x + x^2 + x^3$

$x^3 + x + 1$

$x + x^5 + x^6 + 1$

$x + x^4 + x^2 +$ [illegible]

~~$x + x^6 + x^3 + x^2 + 1$~~

$x^5 + x^4 + x^6 + x^2 + 1$

$x^5 + x^2 + x^3$

$x^4 + x^6 + 1 + x^3$

$x^4 + x^6 + x^3$

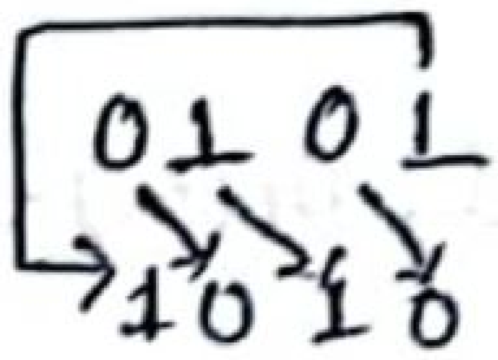
$1 \Rightarrow P(x)$

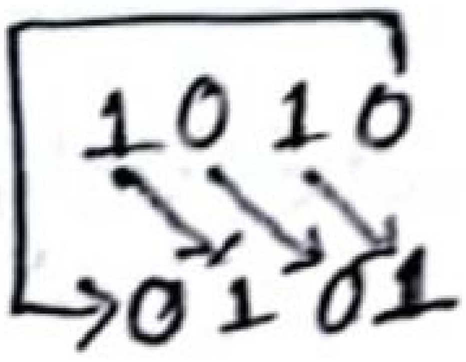
Consider code $\{0000, 0101, 1010, 1111\}$ is it a cyclic code?

$$\Rightarrow \quad \begin{array}{r} 0101 \\ \oplus\ 1010 \\ \hline 1111 \end{array} \qquad \begin{array}{r} 0101 \\ \oplus\ 1111 \\ \hline 1010 \end{array} \qquad \begin{array}{r} 1010 \\ \oplus\ 1111 \\ \hline 0101 \end{array}$$

It follows the property of linearity.

Now, check the property of shifting:-





It follows the property of shifting.

So, given code is cyclic code

Example:-

$V_4 \Rightarrow 4$ tuples

0000
+0101
0101
+1010
1111
1111
+1010
0101

$2^4 = 16$

0000
0001
0010
0011

0100
0101
0110
0111

1000
1001
1010
1011

1100
1101
1110
1111

An example of a subset V_4 that forms a subspace is:-

0000 0101 1010 1111

A set of 2^K n-tuples is called a linear block code if and only if it is a subspace of the vector space V_n of all n-tuples.

P(t) is a pulse, $\tau = T/2$ is the symbol duration.

Another orthogonal waveform set frequently used in communication system is sin x and cos x. $Z_{ij} = \frac{1}{E}\int_0^T s_i(T)\, s_j(t)\, dt = \begin{cases} 1 & \text{for } i=j \\ 0 & \end{cases}$

Analytical representation — waveform representation — vector representation

$S_1(t) = \sin\omega_0 t$

$S_2(t) = -\sin\omega_0 t$

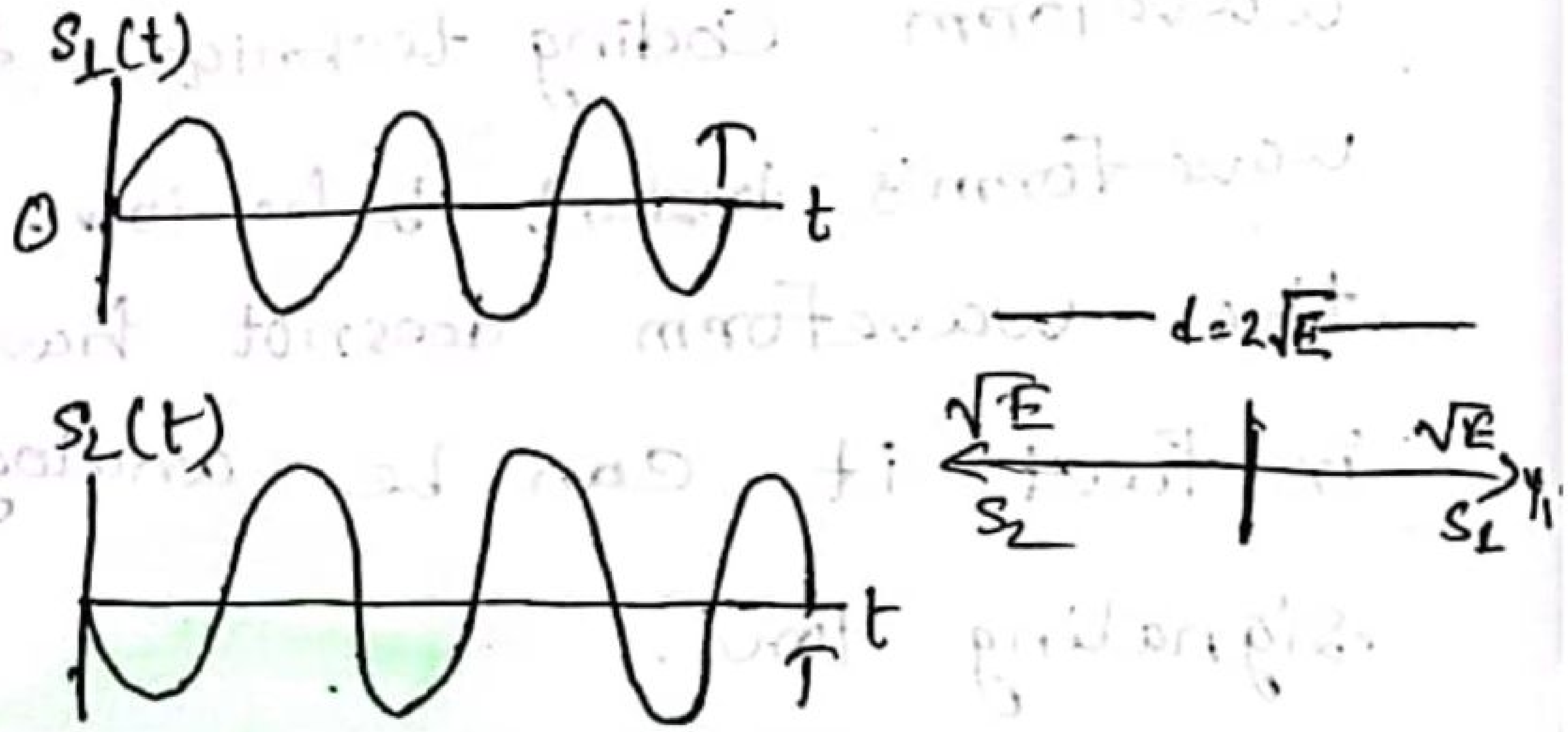


Figure:- Example of an antipodal signal set.

Orthogonal Code :-

⊛ One bit data set can be transformed using orthogonal codewords of two digits each, described by the rows of matrix H_1 as follows:-

Data Set	Orthogonal Codeword Set
0 1	$H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

⊛ To encode 2-bit data set, creating matrix,

H_2

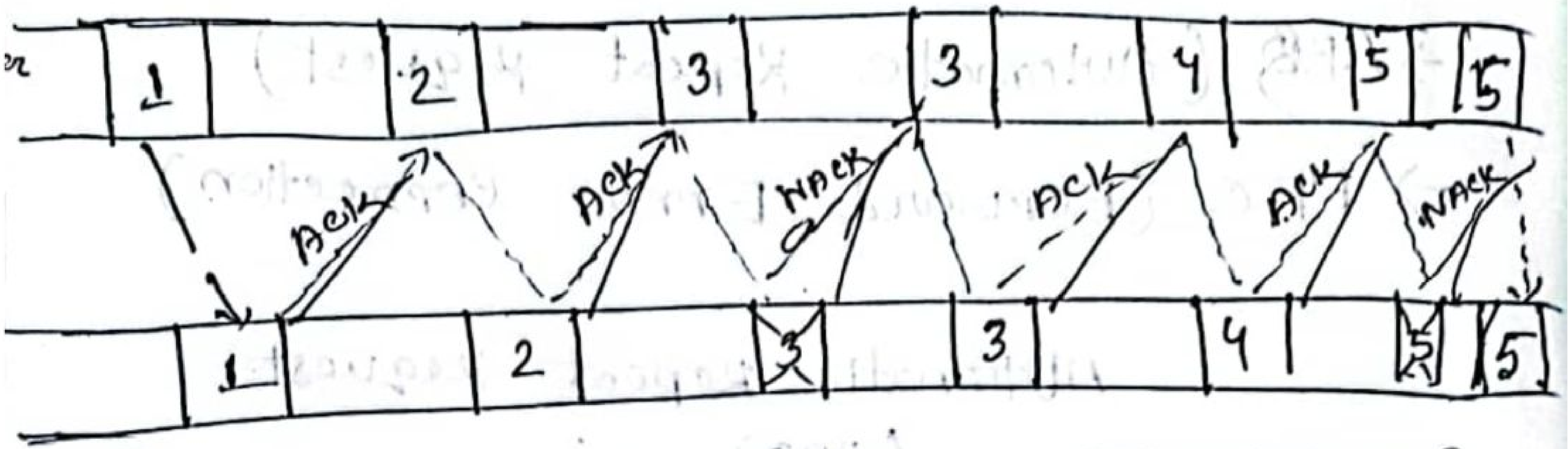
Data Set	Orthogonal Codeword Set
0 0 0 1 1 0 1 1	$H_2 = \begin{bmatrix} 0\,0 & 0\,0 \\ 0\,1 & 0\,1 \\ 0\,0 & 1\,1 \\ 0\,1 & 1\,0 \end{bmatrix}$

$$= \begin{bmatrix} H_1 & H_1 \\ H_1 & \overline{H_1} \end{bmatrix} \Rightarrow$$

ARQ

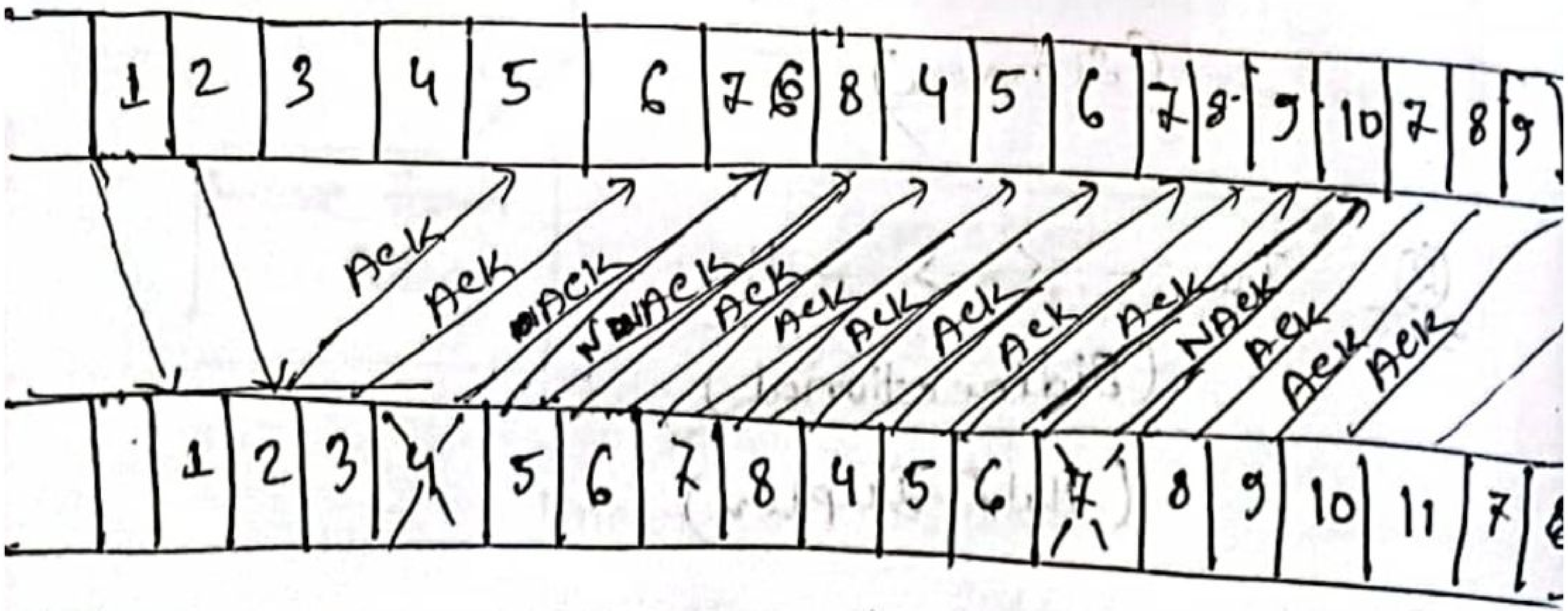
3 types:-

① Stop & wait!-



[এক্ষেত্রে ম্যাসেজ sending থেমে যাবে এবং সেখান হতে আবার সেখান হতেই স্টার্ট হবে)

② Go back N:-



[এক্ষেত্রে Transfer হতেই থাকবে তবে যদি কোনো error আসে তবে সেখান হতেই পুনরায় স্টার্ট হবে]

Hamming Distance:-

101101
110100
× ×   ×

(যেসব value গুলোতে Different

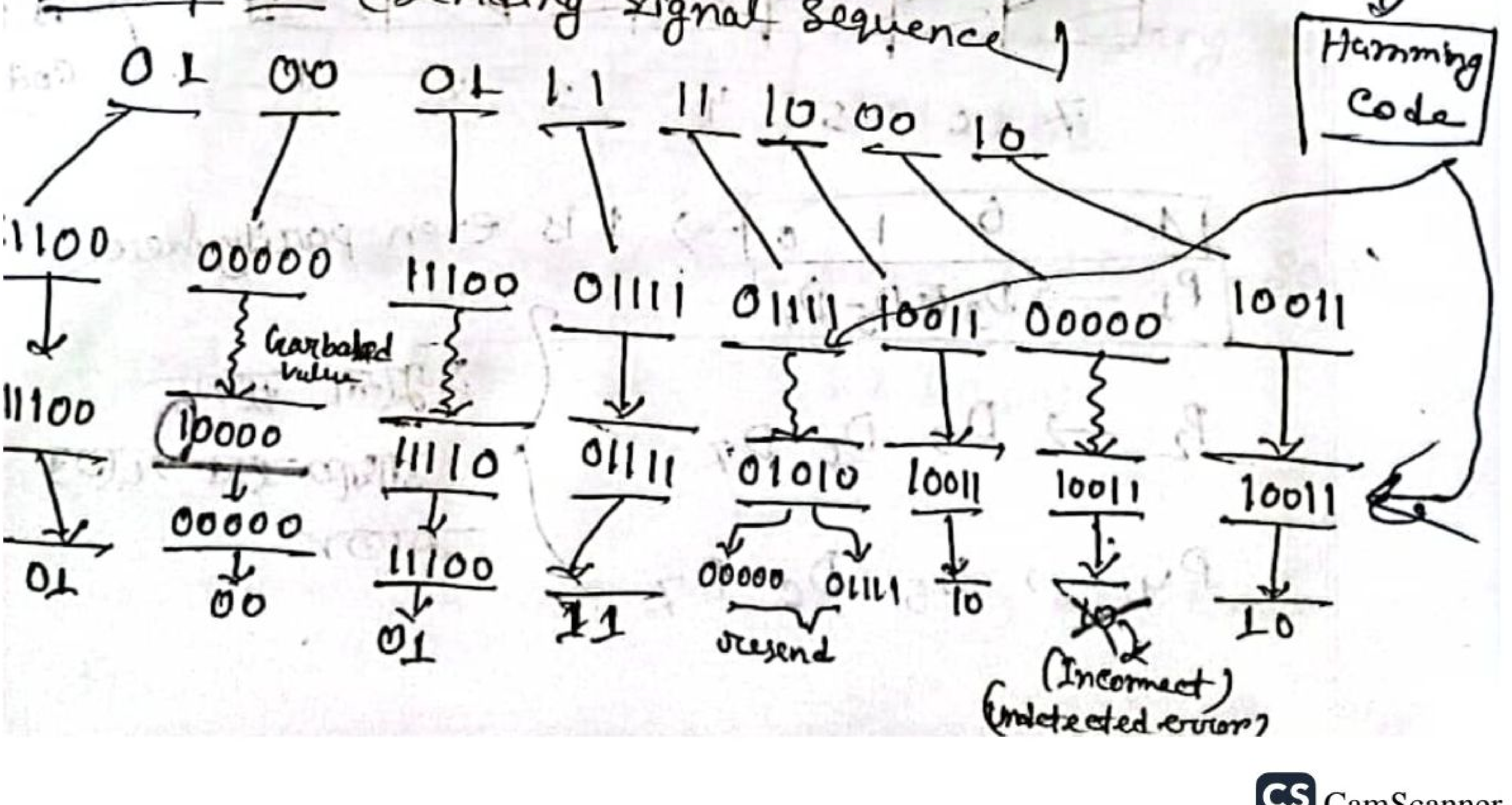
d(101101, 110100) = 3

Error correcting code (ECC)

encoded bit sequence	Corresponding codeword
00	00000
01	11100
10	10011
11	01111

1
2
2
5

Example:- (Sending Signal Sequence)



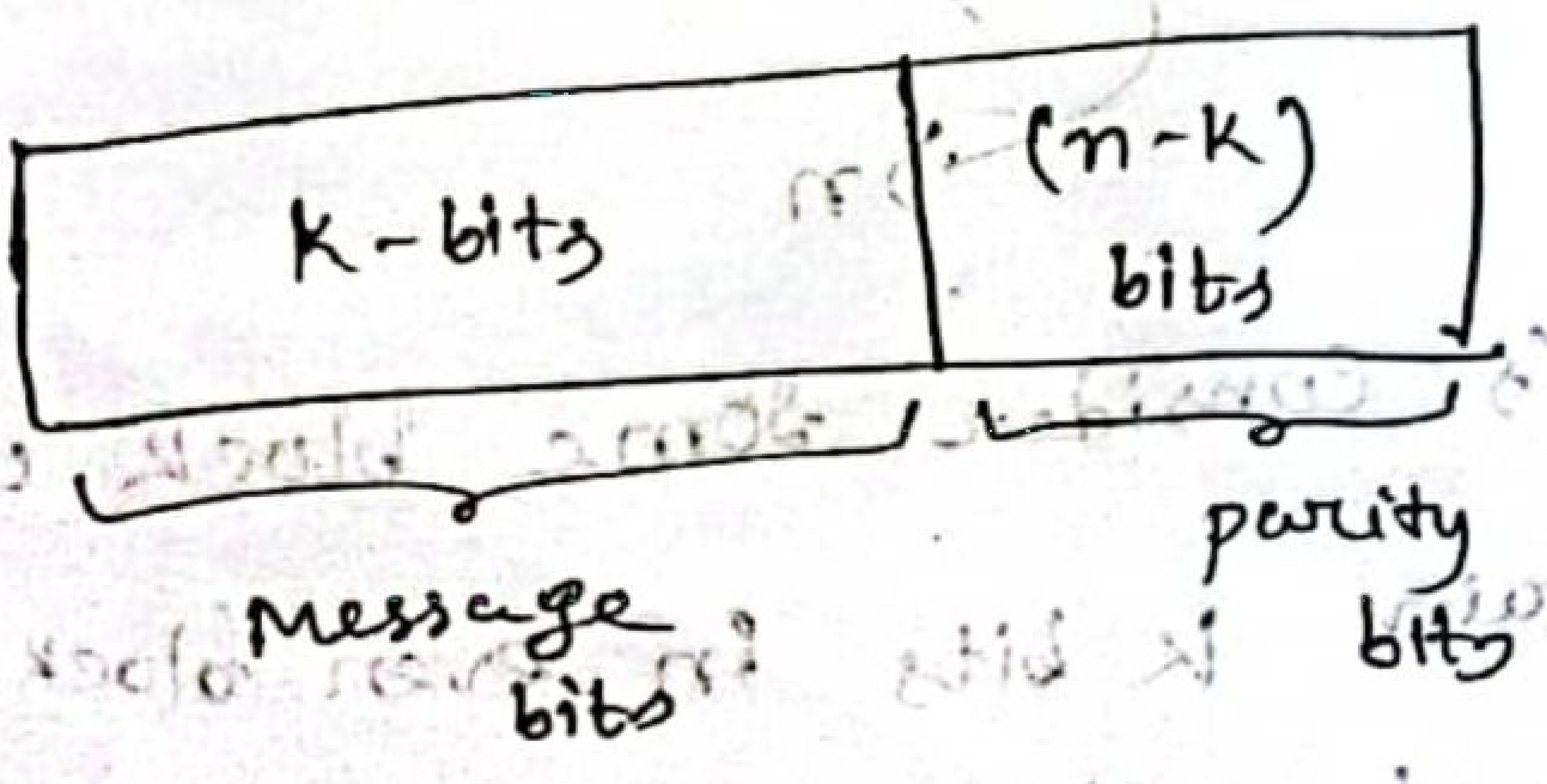
in each block. Here, n is greater than k.

The transmitter adds redundant bits which are $n-k$ bits. The ratio k/n is the code rate. It is denoted by r and the value of r is $r<1$.

The $n-k$ bits are added here, are parity bits.

Any linear block code can be a systematic code, until it is altered. Hence, an unaltered block code is called as systematic code.

Structure of Codeword



Convolution code

The total bits are send = n bits

message bits = k bits

∴ parity bits = n-k bits

1100101	0101011	1010011	21

Example of blocks of data

42	10011010100111 0001010010 111000011101010001

Code rate :-

The redundancy is frequently expressed in terms of the code rate.

The code rate R of a code C of size M and length n is,

$$R = \frac{\log_q M}{n}$$

$M = q^k$, $R = k/n$

The redundancy in ~~ifor~~ information Theory is the number of bits used to transmit a message minus the number of bits of actual information in the message.

$$r = n - \log_q M$$

Code word:- n bit encoded block of bits.

Block length:- The number of bits 'n' after coding is known as block length.

Code Rate:- Ratio of the number of message bits (k) to the total number of bits (n) in a code word.

$$r = k/n$$

Code vectors:-

An 'n' bit code word can be visualized in an n-dimensional space as a vector whose elements or coordinates are bits in the code word. $\begin{bmatrix} 101 \\ 001 \\ 011 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

Hamming distance:-

It is the distance between the two codes expressed in the number of locations in which their respective elements differ.

Hamming weight of a code word [w(x)]:-

Defined as non-zero element's in the code word.

Code efficiency:-

The ratio of message bits to the number of transmitted bits per block. Code efficiency is equal to that of code rate.

Minimum distance (dmin):- It is defined as the smallest hamming distance between any pair of code vectors in the code.

Error:- when the output info doesn't match with input.

Error detecting code:-

whenever a message is transmitted, it may get scrambled by noise or data may get corrupted. So to avoid this, we use error detecting codes.

(Here additional data added to a given digital message)

Example:- parity check.

Error Correcting Codes:-

we can pass some data to figure out the original message from the corrupt message that we received. This type of code is called as an error-correcting code.

⊞ Vector Subspace:-

A subset S of the vector space, V_n is called a subspace of the following two conditions are met.

① All zeros vector is in S.

② The sum of any two vectors in S is also in S.

Suppose, v_i & v_j are two codewords (or codevector) in an (n, k) binary block code. The code is said to be linear if and only if $(v_i \oplus v_j)$ is also a codevector.

[**Note:-**

মোদ্দা কথা হচ্ছে, এখানে যেকোনো দুইটি Codeword যোগ করলে যোগফলটি বাকি Codeword এর মধ্যেই থাকে]]

Example:-

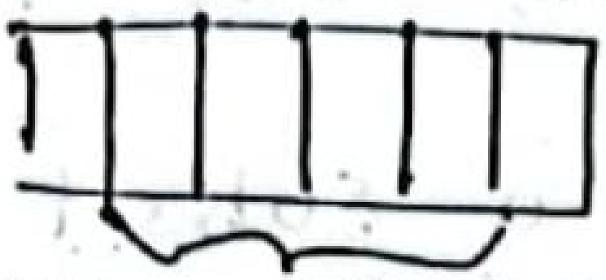
(6,3) Linear Block Code

→ K tuples message vector

$2^K = 2^3 = 8$ message vector and therefo

eight codeword.

There are $2^n = 2^6 = 64$ (6 tuples } in the V_6 vector space)



tuples = 6 টা সদস্যের code word

$p = 6-3$

$P = n-k$

Message Vector	Codewords
000	000000
001	110 100
010	011 010
110	101 110
001	101 001
101	011 101
011	110 011
111	000 111

110 → parity code (Encoding)

100 → Message bi

$= 110100 + 011010$

$= 101110$

$$u_2 = [0\,0\,1]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$= v_3$

$= 101001$

$$u_3 = [0\,1\,0]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$= v_2 = 011010$

- - - .

(To be continued).

∴ Systematic Codevector,

$u = P_1, P_2, P_3 \ldots P_{n-k}$ $\underbrace{m_1, m_2, \ldots m_k}_{\text{Message bit}}$

$$u = [m_1, m_2, m_3] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$\underset{I}{\cancel{P_{n-k}}}$ $\underset{P}{\cancel{I_k}}$

$$= \underbrace{m_1 + m_3}_{u_1} \quad \underbrace{m_1 + m_2}_{u_2} \quad \underbrace{m_2 + m_3}_{u_3} \quad \underbrace{m_1}_{u_4} \quad \underbrace{m_2}_{u_5} \quad \underbrace{m_3}_{u_6}$$

$$\begin{array}{ccc} m_1 & m_2 & m_3 \\ \end{array}$$
$$[1 \ 1 \ 1] = 0 \ 0 \ 0 \ 1 \ 1 \ 1$$

$1 \oplus 1 = 0$

$$[1 \ 0 \ 1] = 0 \ 1 \ 1 \ 1 \ 0 \ 1$$

(ii) $d_{min} \geq 2t+1$ [Here, t = NO. of error correctio

$3 \geq 2t+1$

$\therefore t \leq 1$

$t \leq 1$ means that 2 bit error detected message connected a 1 bit.

(d) Parity check Matrix:-

Theory:-

Parity Generator, $G = [\cancel{P : I_K}]\ [I_k : P]$

(n,k) , $P = n-k$

Parity check Matrix:-

$$\cancel{H = [I_{n-k} : \overset{\text{Transpose}}{P^T}]}$$

$$H = [P^T : I_{n-k}]$$

$$2^r = \begin{bmatrix} 00 \\ 01 \\ 10 \\ 11 \end{bmatrix} = m$$

(e) Determining Transmitted codeword, if received codeword is 100011

We know,

$$S = \overset{\text{received codeword}}{r} . [H^T] \qquad 1\oplus$$

$$= [100011] \begin{bmatrix} 101 \\ 011 \\ 110 \\ 100 \\ 010 \\ 001 \end{bmatrix}$$

$$= [(1\oplus 0\oplus 0\oplus 0\oplus 0\oplus 0)\ (0\oplus 0\oplus 0\oplus 0\oplus 1\oplus 0)\ (1\oplus 0\oplus 0\oplus 0\oplus 0\oplus 1)]$$

$$= [1\ 1\ 0]$$

∴ Transmitted code word = 101011

(Ans)

$$\begin{array}{r} 001110 \\ -000100 \\ \hline 001010 \end{array}$$

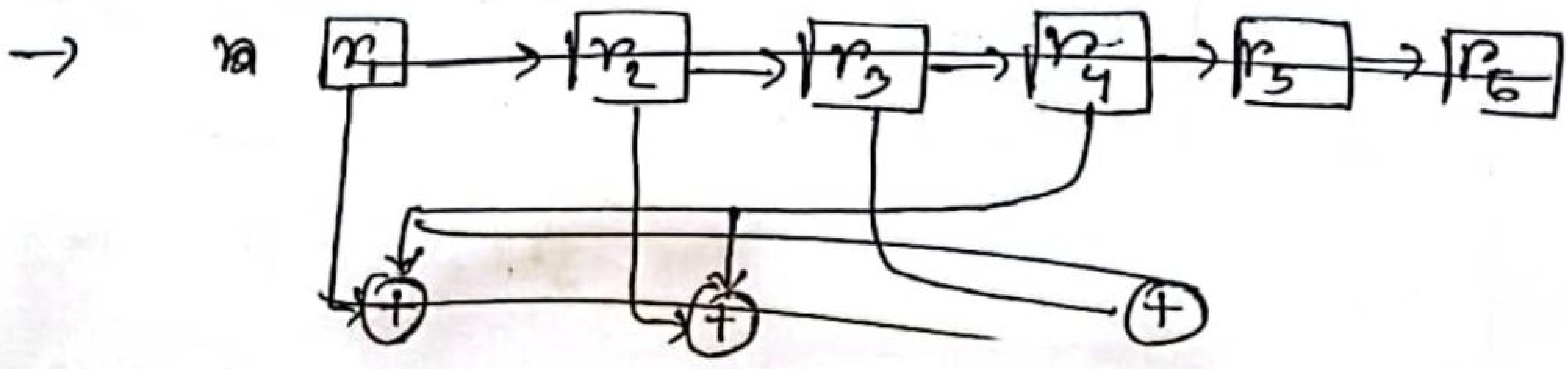
$\therefore S = r\,H^T$

$$= \begin{bmatrix} 0\,0\,1\,1\,1\,0 \end{bmatrix} \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \\ 1&1&0 \\ 0&1&1 \\ 1&0&1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \oplus 1 \oplus, & 1 \oplus 1, & 1 \oplus 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

~~Received vector r,~~



$r = u + e$

$U = \cancel{r \oplus e}\; r + e$ ~~[কারন এটা X-OR অবস্থায় আছে]~~

$= 001110 +$

Syndrome look up Table

Error pattern	Syndrome
000 000	000
000 001	101
000 010	011
000 100	110
— — —	— —

$\boxed{2}$

(6,3) decoder,

$$z \cdot \left[r_1, r_2, r_3, r_4, r_5, r_6 \right] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$S_1 = r_1 + r_4 + r_6$$

$$S_2 = r_2 + r_4 + r_5$$

$$S_3 = r_3 + r_5 + r_6$$

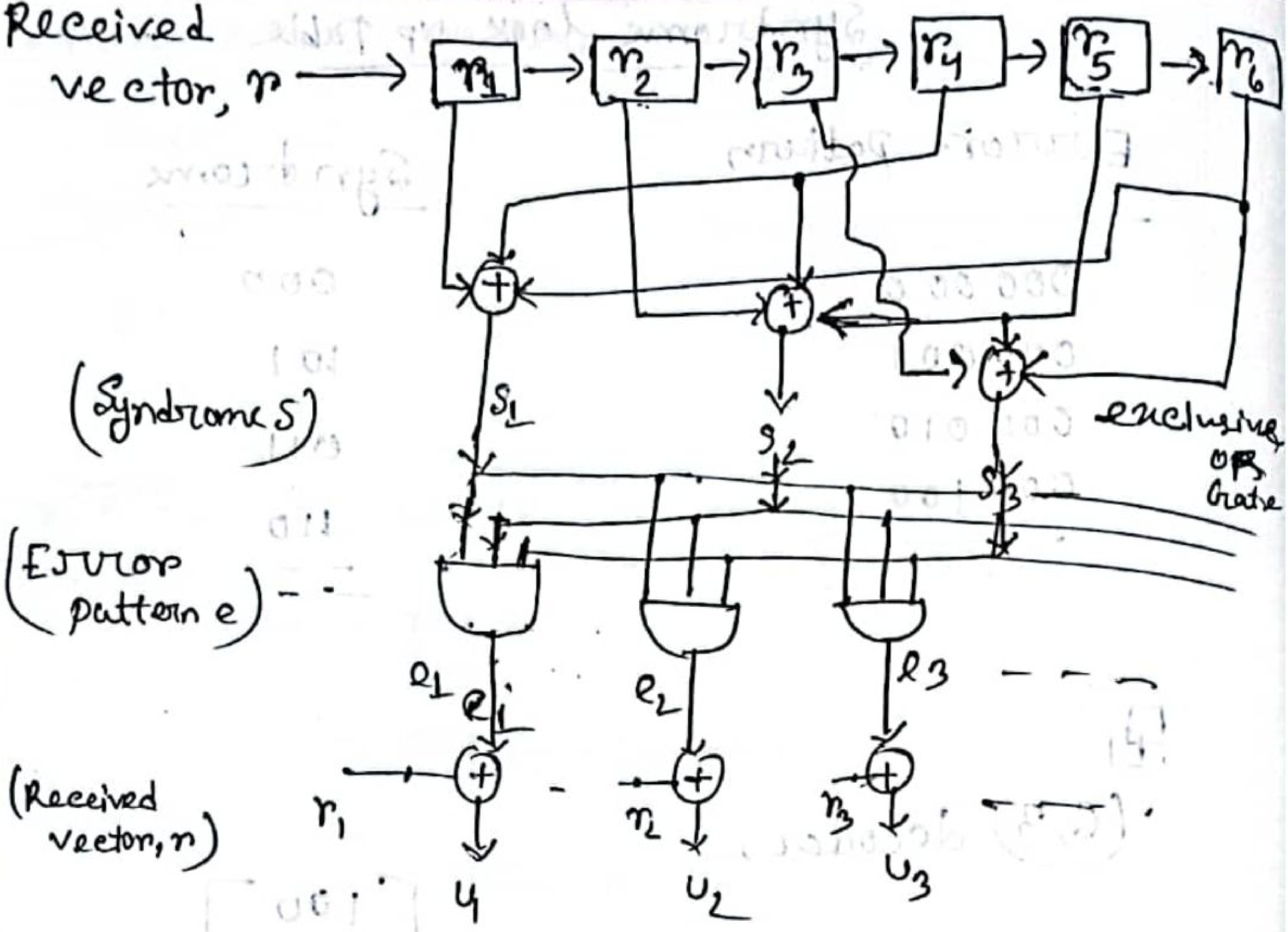


Fig:- Implementation of the (6,3) decoder.

e_1	e_2	e_3	e_4	e_5	e_6	s_1	s_2	s_3
0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1
0	0	0	0	1	0	0	1	1
0	0	0	1	0	0	1	1	0
0	0	1	0	0	0	0	0	1

— — — —

$e_1 = s_1 \bar{s}_2 s_3$

Hamming weight

Hamming weight w(U) of a codeword U is defined to be the number of non-zero elements in U.

=> U = 100101101

w(U) = 5

V = 011110100

Hamming Distance :-

U = 100101101

V = 011110100

⊕

111011001

d(u,v) pair এর মাঝে যখন same হবে না তখন Hamming Distance

=> Hamming Distance = 6

=> $d(u,v) = w(u+v)$ → Hamming weight

That means, The Hamming distance is equal to the Hamming weight of the new codeword

Cyclic shift of a codevector

(*)
Let, $U = 1101$ for $n=4$

Enpress the codeword in polynomial form. Solves for the third end around shift of the codeword

---

Theory:

Suppose, we have a bit = 101 => 5 in decimal

So, Bit to Decimal Deci

$$1\times2^2+1\times2^1+1\times2^0 = 5$$

Here, we will use x in the place of 2^n,

Solution:-

$U = 1101$

$$U(x) = 1\times x^0 + 1\times x^1 + 0\times x^2 + 1\times x^3$$

$$= 1 + x + x^3$$

7-bit codeword = $x^3(1+x^2+x^3)+1$

$= x^3 + x^5 + x^6 + 1$

~~$= 1 + x^3 + x^5 + x^6 \quad x^6$~~

~~$= 0\,1\,0\,0\,1\,0 + 1$~~

~~$= 1001011$~~ $= [1101001]$

এখানে ম্যাসেজ vector লেখার সময় বড় হতে ছোট

Systematic Block Code:-

$U = (P_1, P_2, P_3, \ldots P_{n-k-1}, m_0, m_1, \ldots m_{k-1})$

Generator polynomial; $G(x) = 1 + x + x^3$

generator a systematic codeword for (7,4) codeword for the message vector, $m = 1011$

=)

Solution:-

$\therefore m = 1011$

$M(x) = 1 + x^2 + x^3$

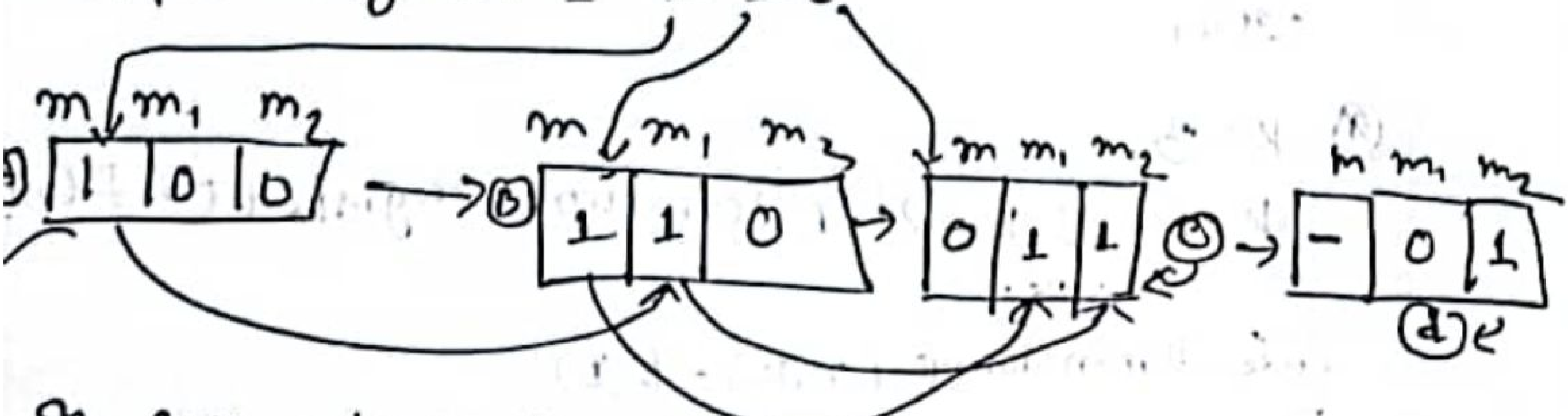
$G(x) = 1 + x + x^3$

parity, $p = 7 - 4 = 3 = 1 + x + x^2$

Code tree:-

→ each branch of tree represent an I/p symbol with corresponding pair of output binary symbols indicating on the branch.

⇒ Input msg bit = 1 1 0



So, our output for for convolution code,

(A), $x_1 = m \oplus m_1 \oplus m_2 = 1 \oplus 0 \oplus 0 = 1$

$x_2 = m \oplus m_2 = 1 \oplus 0 = 1$

output $x_1x_2 = 11$

For (B),

$x_1 = 1 \oplus 1 \oplus 0 = 0$

$x_2 = 1 \oplus 0 = 01$

$\rightarrow x_1x_2 = 01$

For c,

$x_1 = 0 \oplus 1 \oplus 1 = 0$

$x_2 = 0 \oplus 1 = 1$

$x_1x_2 = 01$

Code Tree

we know,

m_1	m_2	State
0	0	a(00)
0	1	b(01)
1	0	c(10)
1	1	d(11)

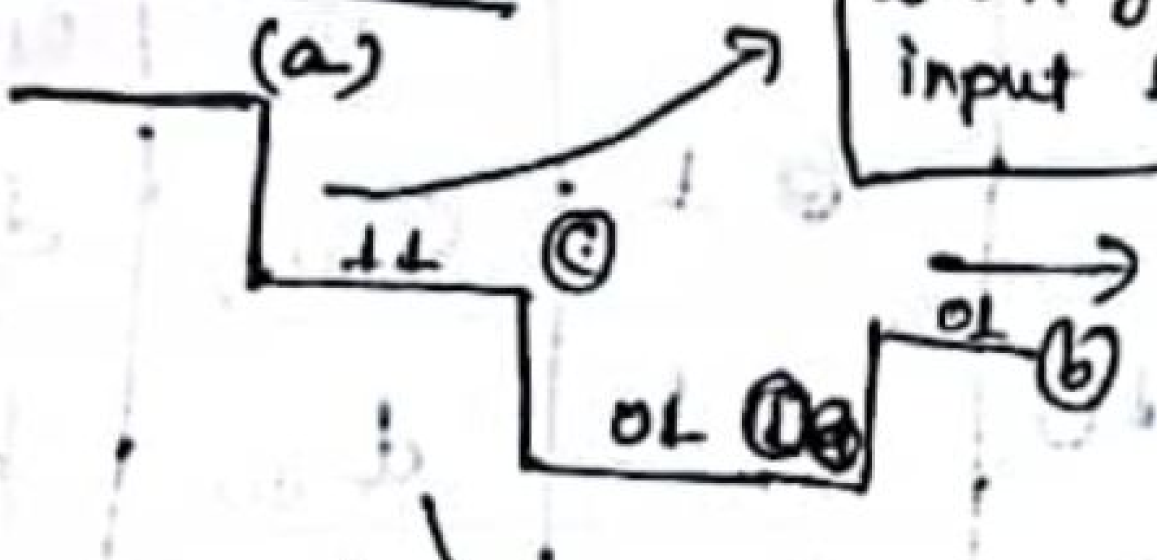
So, A have state-a as $m_1=0$ $m_2=0$

B " State-c as $m_1=1$ $m_2'=0$

C " State-d $m_1=1$ $m_2=1$

D " State-b $m_1=0$ $m_2'=1$

State-a



When you give input 1 goto down step

Note:-
While making it we will see m bit only

→ up step means input = 0

Down " " " = 1

(Code tree)

$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

⊛ Then using this we will have to draw code trellis.

While drawing need to remember couple of things:-

⊛ Solid lines ("———") explains input bit 0.

⊛ Dash lines ("-----") " " " 1

[এটা বুঝানোর Trellis দ্বারা হবে মাত্র] ~~State Diagram~~

Trellis Diagram

Example:–

For the encoder shown in figure shown in figure Show the state changes & the resulting Output codeword sequence U for the message sequence $m = 11011$ · Followed by, $K - 1 = 2$ zeros to flush the register. Assume that the initial contents of the registers are zeros.

Solve:–

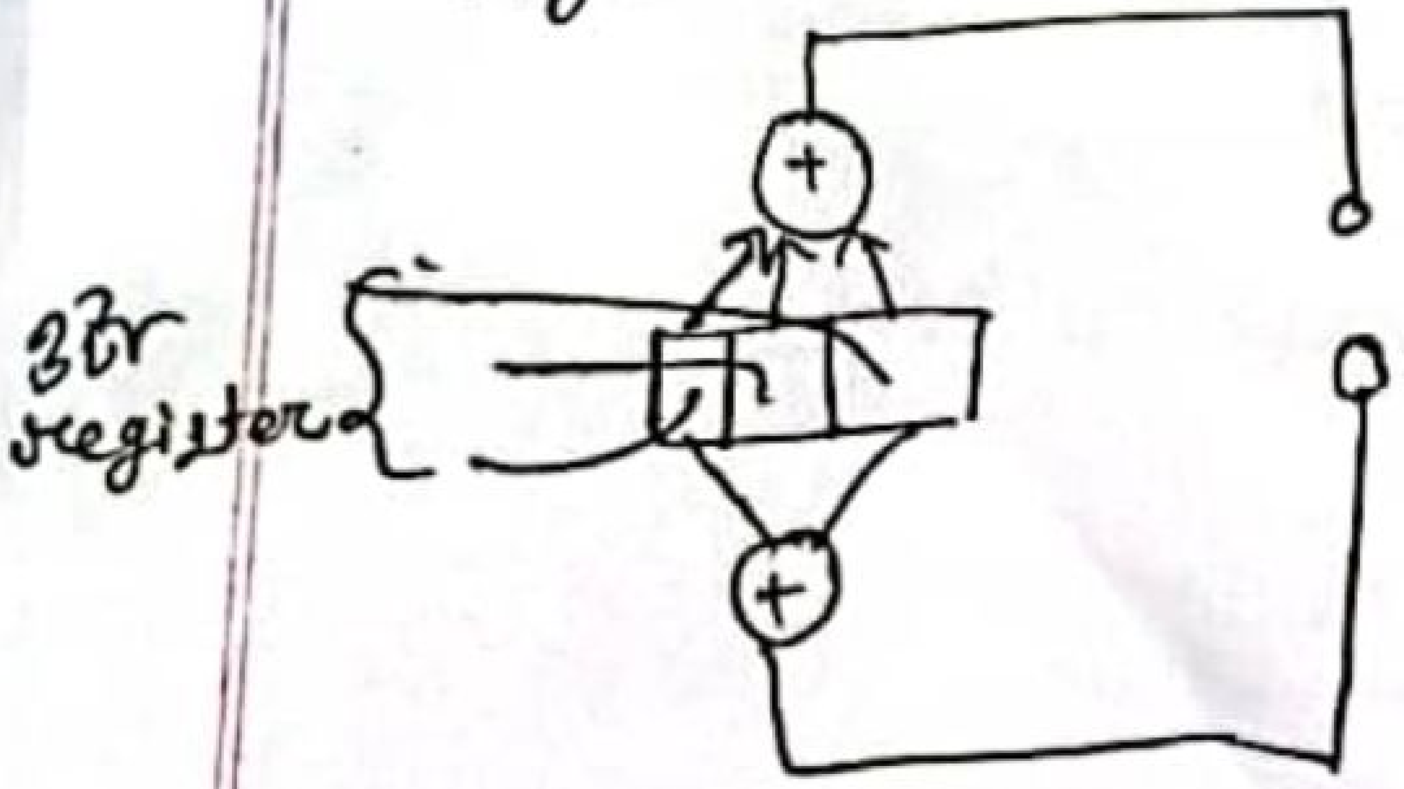
$$u_1 = m_1 \oplus m_2 \oplus m_3$$

$$u_2 = m_1 \oplus m_3$$

$$m = 11011$$

Current State =

Figure



Code Tellis & State Diagram of Convolutional codes

⊛ First we need to calculate all possible state :-

m	→ m_1	→ m_2	x_1	x_2	Current State	Next State
0	0	0	0	0	a 00	a 00
1	0	0	1	1	a 00	c 10
0	0	1	1	1	b 01	a 00
1	0	1	0	0	b 01	c 10
0	1	0	1	0	c 10	b 01
1	1	0	0	1	c (10)	d (11)
0	1	1	0	1	d (11)	b (01)
1	1	1	1	0	d (11)	d (11)

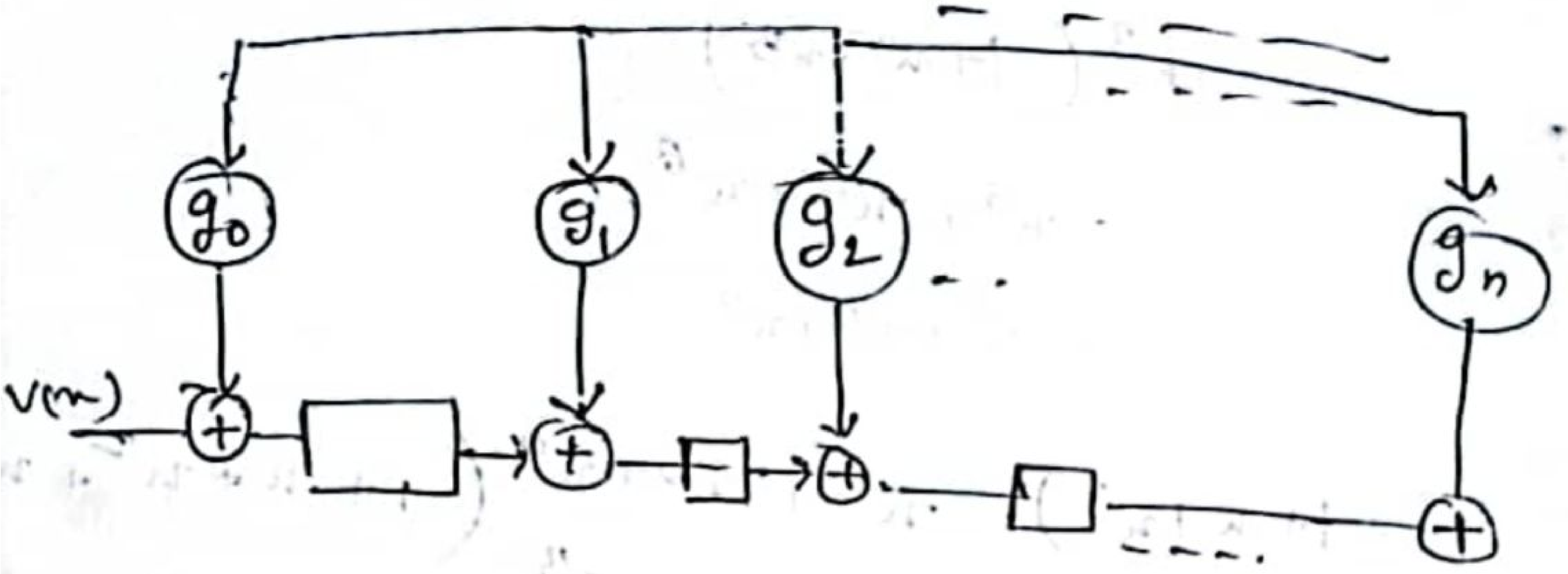
⊛ First write down all possible table for both 0 & 1.

⊛ For current state use bits m_1 & m_2

⊛ For next state count for m & m_1

Circuit Dividing polynomial

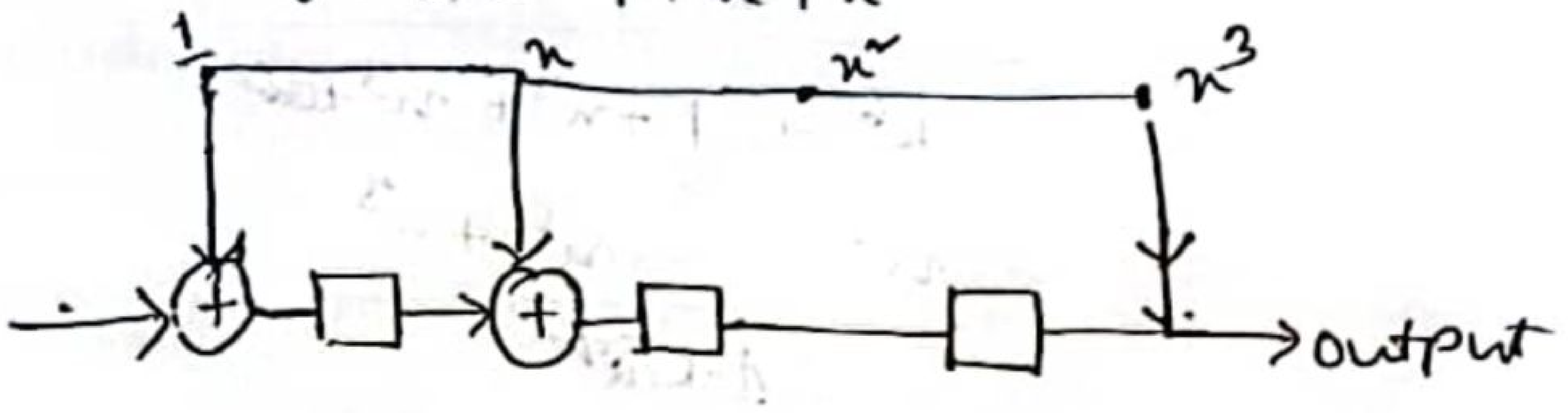
model



$V(x) = x^6 + x^5 + x^3$
$g(x) = 1 + x + x^3$ } so draw the circuit

Solve:-

$g(x) = 1 + x + x^3$



Figure

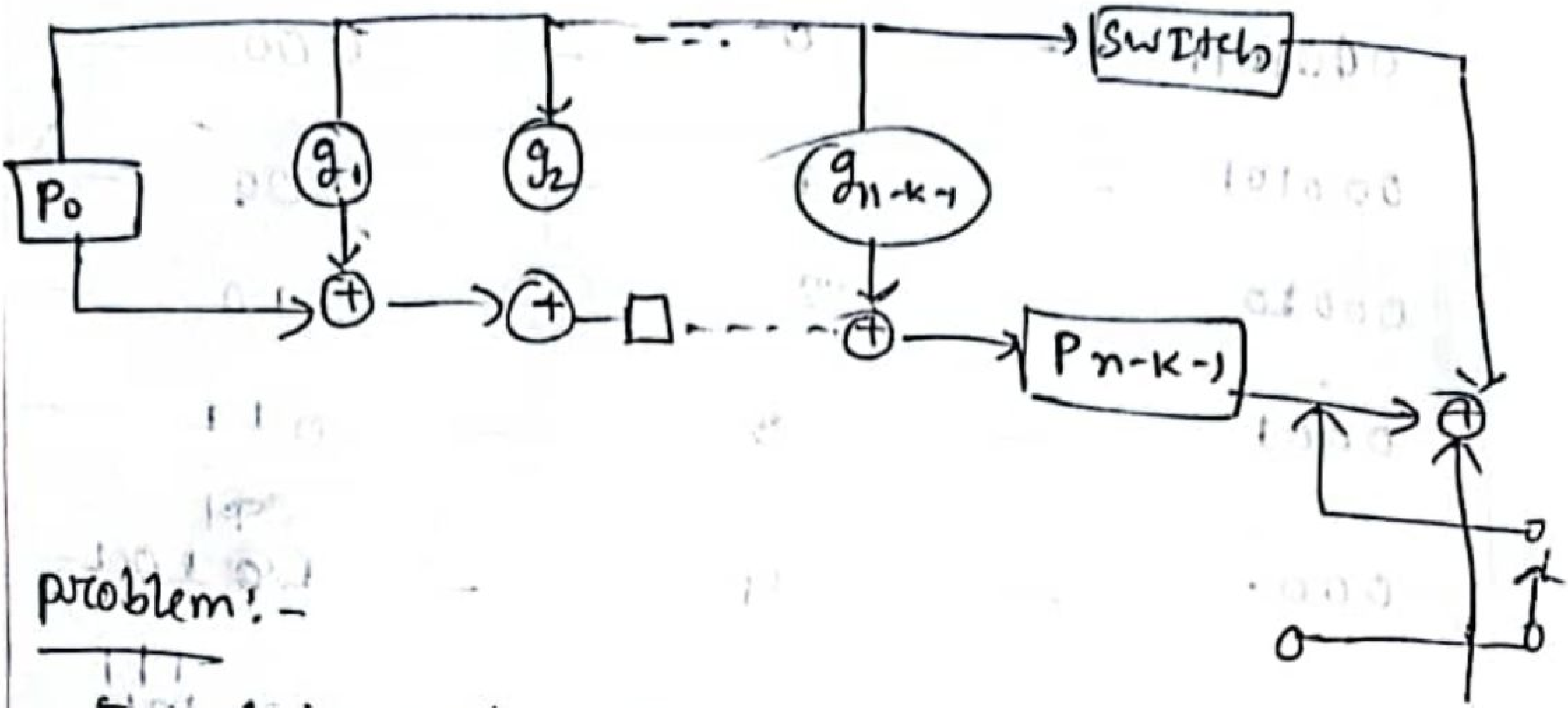
Here, $V(x) = x^6 + x^5 + x^3$

= ~~1101000~~

= 0001011

$x^3 = 0$
$x^2 = 0$
$x = 0$
$1 = 0$

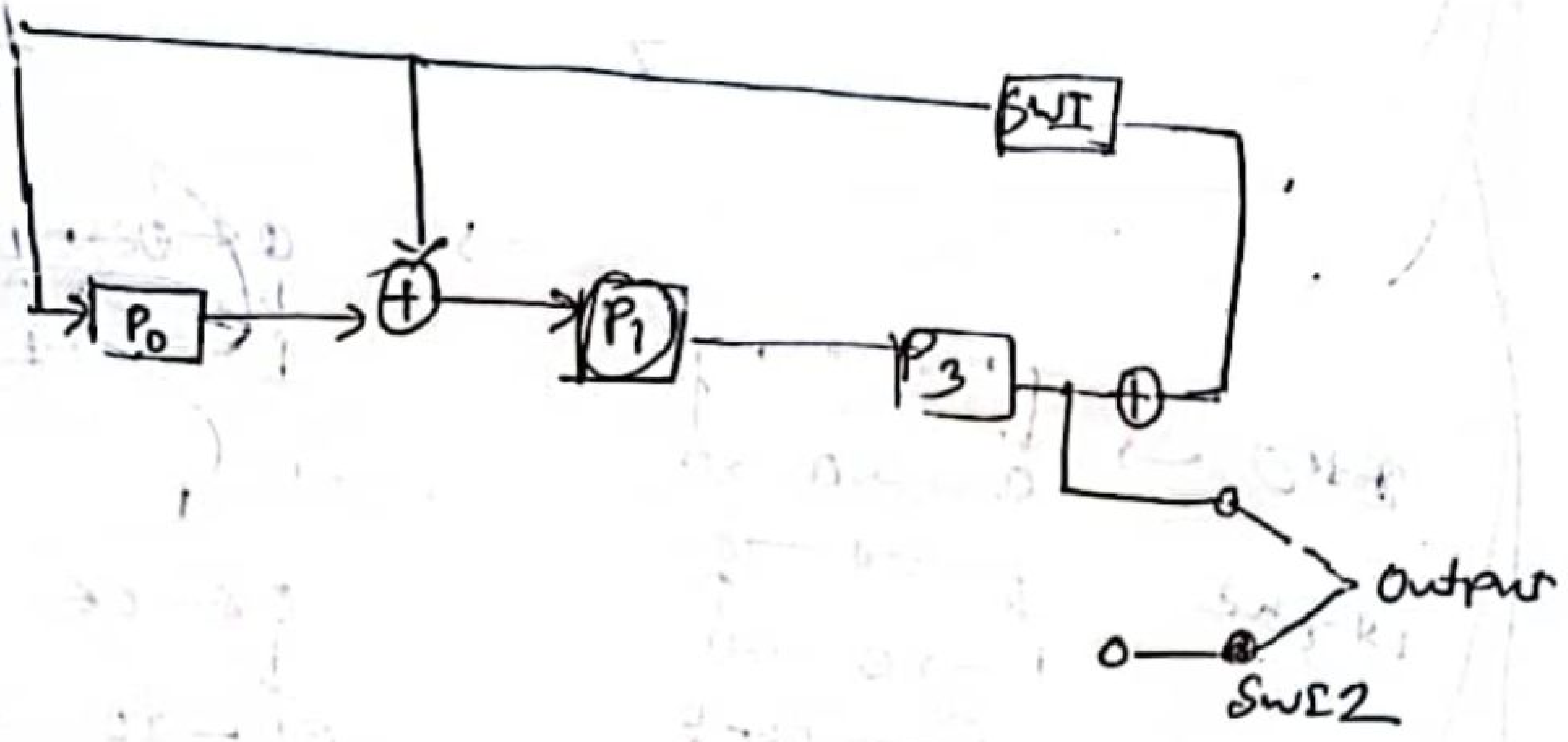
Codeword:- $P(x) + m(x)$



problem:-

(7,4) codeword $m = 1011$

$g(x) = 1+x+x^3$



Divide x^{n-k} of $m(x)$ by $g(x)$ using polynomial

$$x^3(1+x^2+x^3)$$

$$= x^3+x^5+x^6$$

$$= x^6+x^5+x^3$$

$$1+x+x^3)\; x^6+x^5+x^3 \;(1+x+x^2+x^3$$

$$\underline{1+x \quad x^3}$$

$$x^6+x^5+1+x$$

$$\underline{x^2+x^4 \quad +x}$$

$$x^6+x^5+1+x^4+x^2$$

$$\underline{x^5 \quad x^3+x^2}$$

$$x^6+1+x^4+x^3$$ [illegible]

$$\underline{x^6 \quad x^4+x^3}$$

1 [illegible]

∴ [illegible] = [illegible]

$$\therefore v(x) = P(x) + m(x)$$

$$= \underbrace{100}_{P}\underbrace{1011}_{mt}$$

NOW, ↗ i= Number of shifting

$$x^i \cdot U(x) = x^3(1+x+x^3)$$

$$= x^3+x^4+x^6 \quad [\text{where } i=3]$$

[Note:- কোনো Codeword কে Generator Matrix দিয়ে divide করলে New codeword পাওয়া যায়]

☐ Divide, $x^1 U(x)$ log x^n $= G = x^4+1$

$$x^4+1 \,)\, x^3+x^4+x^6 \,(\, 1+x^2$$

$$x^4 \qquad +1$$

$$\overline{x^3+x^6+1}$$

$$\cancel{x^8}\; x^6+x^2$$

$$\overline{x^3+1+x^2}$$

∴ Reminder: $1+x^2+x^3$ Ans.

So, from polynomial to Bit

$U^3 = 0\,1\,0\,1\,1$

⊛ Consider, Message signal, $m = [1\underbrace{011}_{4}]$

So the message polynomial, =

$$M(x) = 1 + x^2 + x^3$$

Genarator polynomial $g(x) = x^3 + x + 1$

~~we know that~~

Determine 7 bit Codeword for systematic code.

Solve:-

$$m(x) = (1 + x^2 + x^3)$$

$$G(x) = (x^3 + x + 1)$$

we know that,

7 bit Codeword = $x^{n-k} \cdot m(x) + P(x)$

we know that

$$P(x) = \text{Remainder of } \frac{x^{n-k}(m(x))}{G(x)}$$

Here,

~~$m(x) = 1 + x^2 + x^3$~~

$$= \text{Remainder of } \frac{x^{7-4} \cdot m(x)}{G(x)}$$

$$= \frac{x^3(1 + x^2 + x^3)}{x^3 + x + 1}$$

properties of cyclic code

Cyclic code : Subclass of a linear block codes where cyclic shift in the bits of the codeword results in another codeword. (অর্থাৎ, linear Block code এ যদি cyclic code use করে আবর্তনটি code তৈরি করা হয় তবে সেই codeword ও অন্য যেকোনো codeword এ পাওয়া যাবে।

=) It is widely used in satellite communication as the information sent digitally encoded & decoded using cyclic coding. ~~These are~~

These are error correcting codes where the actual information is sent over the channel by combining with the parity bits. Various other important codes like, Reed Solomon, Golay, Hamming, BCH, etc can be represented using cyclic codes.

property ~~of~~ 02 :- property of cyclic shifting.

After a left or right shift in the bits of codewords the resultant code generated must be another codeword.

Suppose,

C is a codeword given as:

$$C = [c_1, c_2, \text{----} \; c_{n-1}]$$

Then after cyclic shifts,

$$C = [c_1 \; c_2 \; c_3 \; \text{---} \; c_{n-1}]$$

$$\text{Right shift } C_0 = [c_{n-1} \; c_1 \; c_2 \; c_3 \; \text{---} \; c_{n-2}]$$

$$C^1 = [c_{n-2} \; c_{n-1} \; c_1 \; c_2 \; c_3 \; \text{---} \; c_{n-3}]$$

Example:-

110 : right shift will provide: 011

101 : " " " " 110

011 : " " " " 101

Encoding

Non systematic cyclic Encoding:-

Consider message signal given as,

$m = [1\,1\,1\,0]$

$M(x) = 1 \times x^3 + 1 \times x^2 + 1 \times x^1 + 0 \times x^0$

$= x^3 + x^2 + x \cancel{+1}$

Here, Generate polynomial, $G(x) = x^3 + x + 1$

Non-Systematic code, codeword,

$C(x) = M(x) \cdot G(x)$

$\therefore C(x) = (x^3 + x^2 + x \cancel{+1})(x^3 + x + 1)$

$= x^6 + x^5 + x^4 \cancel{+x^3} + x^4 + x^3 + x^2 \cancel{+x}$
$+ x^3 + x^2 + x \cancel{+1}$

Here, duplicate bit's addition will result $\overset{to}{}$ 0

So, $C(x) = x^6 + x^5 + x$

Hence, from the above Codeword polynomial, the codeword will be :-

$$C = [\,1\,1\,0\,0\,0\,1\,0\,]$$

Systematic cyclic Encoding

Message Signal, M = [1011]

polynomial $\longrightarrow x^{3}\times 1 + x^{2}\times 0 + x^{1}\times 1 + x^{0}\times 1$

= ~~x^4+x^2+x~~

$= x^3 + x + 1$

Generator polynomial, $G(x) = x^3+x+1$

The eqn for determining 7 bits codeword for systematic code is given as: $C(x) = x^{n-k}.\,M(x)+P(x)$

$\Rightarrow$ [illegible] = [illegible] × [illegible] + [illegible]

P(x) represents the parity polynomial and is given by, $P(x)$ = Remainder of $\left\{\dfrac{x^{n-k}\cdot M(x)}{G(x)}\right\}$

Example:-

Construct a systematic cyclic codes (7,4) using generator polynomials $g(x) = x^3+x+1$ with message (1011)

~~$x^3+x+1)$~~ So

Solve:-

Message code = 1011

in polynomial = $x^3 + x + 1$

~~$x^3+x+1)\ x^3$~~

Here,

x^{n-k} of $m(x) = x^{7-4} = x^3\,(x^3+x+1)$

$= x^6 + x^4 + x^3$

Now need to divide it with $g(x)$

$$x^3+x+1 \overline{)\, x^6+x^4+x^3 \,}\ x^3$$

$$x^6+x^4+x^3$$

$$\overline{\qquad\qquad\qquad 0 \qquad\qquad}$$

$G = [I \; P]$

$H = [I \; P^T]$

$$G = \begin{bmatrix} 110 & 100 \\ 011 & 010 \\ 101 & 001 \end{bmatrix}$$

$$S = rH^T = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$H = [I_{n-k} \; P^T]$$

$$= [I_m \; P^T]$$

~~$= [1\,00\,101$~~

~~$= [1\,0\,0]$~~

S ~~$= e \cdot$~~ H^T

$$H = [I_3 \; P^T] = \begin{bmatrix} 100 & 101 \\ 010 & 110 \\ 001 & 011 \end{bmatrix}$$

Another example:-

1 0 1 1

1	0	1	P4	1	P2	P1
▨	▨	▨	0	▨	0	1
D7	D6	D5		D3		

P_1 will depend on

→ $D_3 D_5 D_7$

1 $\boxed{1 \ 1 \ 1}$ so 1 is Add here but we need to make it even so another 1 is needed so $P_1 = 1$

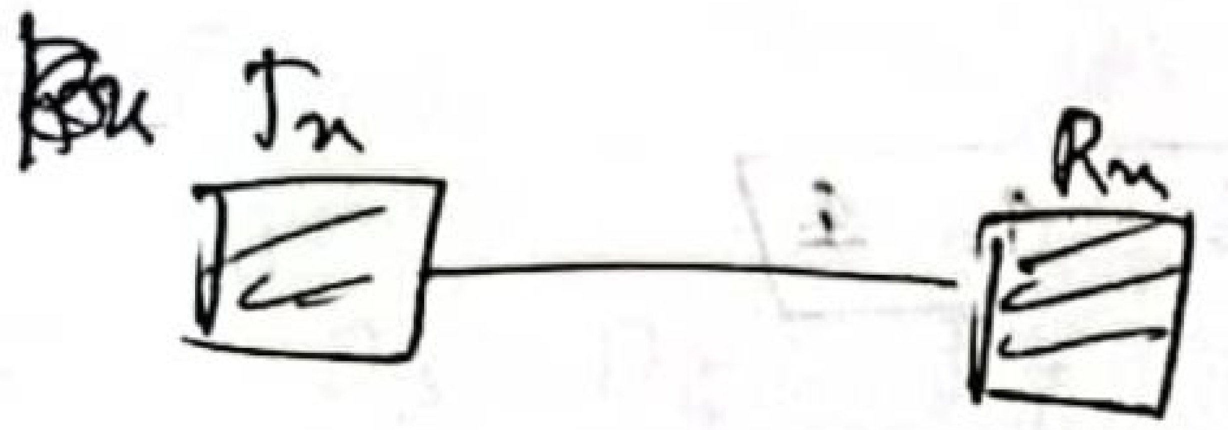
$P_2 = D_3 \ D_6 \ D_7$

0 $\boxed{1 \ 0 \ 1}$

$P_3 = D_5 \ D_6 \ D_7$

0 $\boxed{1 \ 0 \ 1}$

Transmitter & Receiver:-



we are going to send 1010101

~~The~~ But there noise has been added. So the signal is changed to: 1110101

Solve:-

The receiver will see the parity bits first so,

☐ Channel coding :-

It can be partitioned into two study areas, waveform coding and structured sequence. (Add an extra bit with message bit to detec the error as well as correct the error)

Waveform coding

Waveform coding techniques describe the waveform's instant behavior. This means that the waveform doesnot have to be speech in fact it can be analog data or a signaling tone.

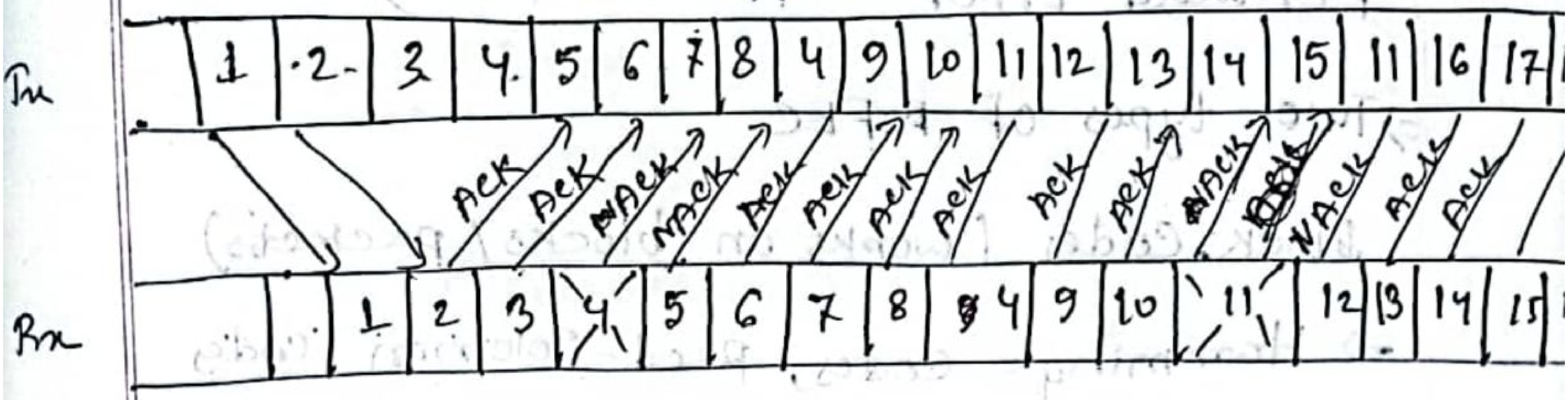
⊛ Antipodal and orthogonal signals :-

An orthogonal signal set made up of pulse waveforms,

$$s_1(t) = p(t) \qquad 0 \leq 1 \leq t$$

$$s_2(t) = p\left(t - \frac{T}{2}\right) \qquad 0 \leq 1 \leq T$$

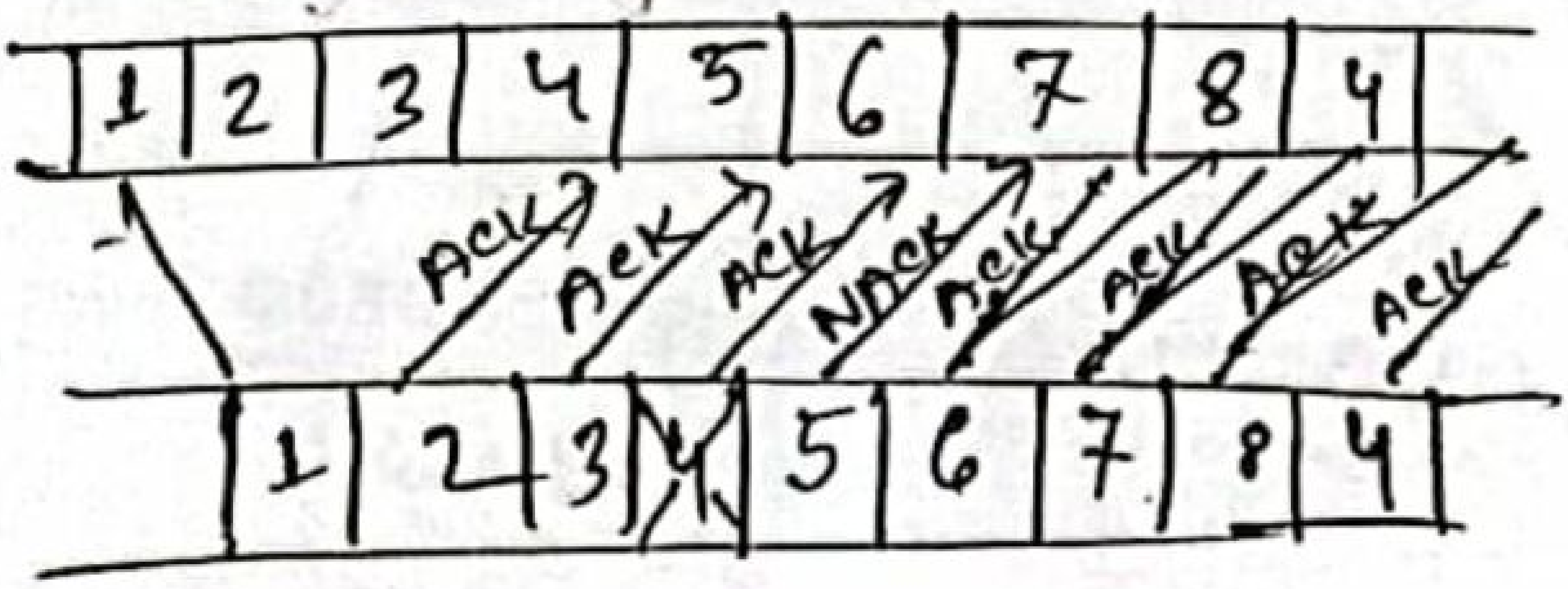
③ Selective & Repeat:-



[যেটা আসবে না সেখানে Not Acknowledgement signal পাঠাবে। এবং শুধুমাত্র ঐ জিনিসটাই পুনরায় পাঠাবে।]

Note:-

✱ এখানে ACK NACK দুটো signal হতে যাবে, অর্থাৎ,



FEC(Single channel):-

Forward Error Correction (FEC)

⇒ Two types of ~~F~~ FEC

Block codes (works on blocks/packets)

→ Hamming codes, Reed-Solomon codes

→ Convolutional codes (Arbitrary length symbols/bits)

uses:-

→ Reverse channel available না থাকলে

→ Retransmission covenient না হলে।

→ Error is too much high হলে।

$$\begin{array}{ccccccc} 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ \hline D_7 & D_6 & D_5 & P_4 & D_3 & P_2 & P_1 \end{array}$$

$P_1 = D_3 \quad D_5 \quad D_7$

$= 1 \boxed{1 \quad 1 \quad 1}$

$\therefore P_1 = 1$

$P_2 = D_3 \quad D_6 \quad D_7$

$= 1 \boxed{1 \quad 1 \quad 1}$.

$P_2 = 1 \Rightarrow$ 1 হবে কিন্তু error এর জন্য signal 0 হয়ে যাচ্ছে।

$P_4 =$ ~~D_{34} D_5~~ $D_5 \quad D_6 \quad D_7$

$= 1 \boxed{1 \quad 1 \quad 1}$

$\therefore P_4 = 1 \Rightarrow$ কিন্তু error থাকায় signal 0 হয়ে আছে।

Hamming Code-Error Correction

Ex — If the 7-bit hamming code word received by a receiver is $\boxed{1011011}$. Assuming the even parity state whether the received code word is correct or wrong. If wo wrong locate the bit having error.

Solution :-

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	0	1	1	0	1	1

$P_4 = \boxed{1 \quad \overset{D_5}{1} \quad \overset{D_6}{0} \quad \overset{D_7}{1}}$ → [odd থাকায় যা error]

→ $P_4 = 1$ (There is an error)

$P_2 = D_3 \quad D_6 \quad D_7$

$= 1 \quad 0 \quad 0 \quad 1$ → even (So there is no error

$P_2 = 0$ (As there is no error or contradiction)

$P_1 = D_3 \quad D_5 \quad D_7$

$= \boxed{1 \quad 0 \quad 1 \quad 1}$ ⇒ [3-টা 1 আছে যা odd তাই error]

(ay)

$$G = \begin{bmatrix} 100 & \overbrace{101}^{\to \text{ P or parity bit}} \\ 010 & 011 \\ 001 & 110 \end{bmatrix}$$

Finding all corresponding code vectors

We know,

$$G = [\overset{\to 3\text{bit}}{I} : \overset{\to 3 \text{ bit}}{P}]$$, ~~$G = [P\ P]$~~

Here,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Code bits, $C = n - K$

$= 6 - 3 = 3$ bit

Message bit, $M = 2^K = 2^3 = 8$ bit   c=4

Now, $[C] = [m][P]$

$$[C_0, C_1\ C_2] = [M_0 M_1 M_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$C_0 = M_0 \oplus M_2$, $C_1 = M_1 \oplus M_2$, $C_2 = M_0 \oplus M_1$

M_0	M_1	M_2	C_0 $(M_0 \oplus M_2)$	C_1 $(M_1 \oplus M_2)$	C_2 $(M_0 \oplus M_1)$
0	0	0	0	0	0
0	0	1	1	1	0
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	0	0

(b)

∴ Minimum Humming distance,

M_0	M_1	M_2	C_0	C_1	C_2	Minimum H.D →
0	0	0	0	0	0	→ 0
0	0	1	1	1	0	→ 3
0	1	0	0	1	1	→ 3
0	1	1	1	0	1	→ 4
1	0	0	1	0	1	→ 3
1	0	1	0	1	1	→ 4
1	1	0	1	1	0	→ 4
1	1	1	0	0	0	→

Syndrome look up Table

Error pattern (e)	Syndrome
000000	000
000001	101
000010	011
000100	110

Decoder

(6,3) [illegible]

$S = r\,H^T$

$$= [r_1, r_2, r_3, r_4, r_5, r_6]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$s_1 = r_1 + r_4 + r_6$

$s_2 = r_2 + r_4 + r_5$

$s_3 = r_3 + r_5 + r_6$

110

$$\begin{array}{r} 100011 \\ \oplus\,000110 \\ \hline 1000101 \end{array}$$

Syndrome Testing

~~Step.~~

Problem:-

Suppose a received vector $r = 001110$ is received and find the syndrome vector value, $S = r \cdot H^T$

And verify that is equal to eH^T

where,

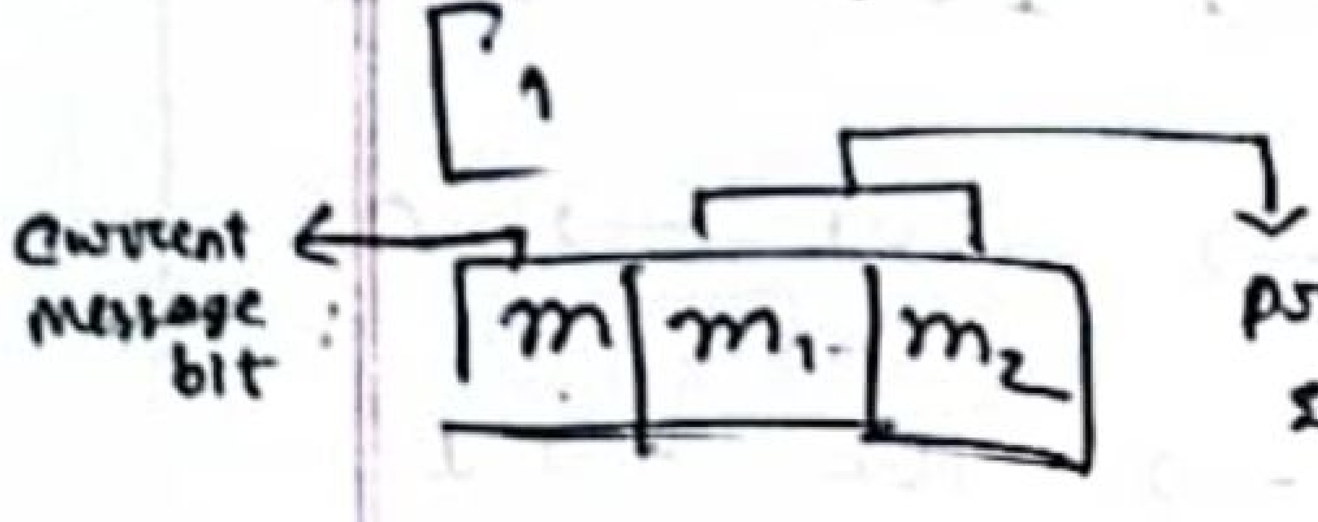
$$G = \begin{array}{c} \begin{array}{cc} P & 3=R \end{array} \\ \begin{bmatrix} 1\,1\,0 & 1\,0\,0 \\ 0\,1\,1 & 0\,1\,0 \\ 1\,0\,1 & 0\,0\,1 \end{bmatrix} \end{array}$$

Solution:-

$S = rH^T$

Convolution codes:-

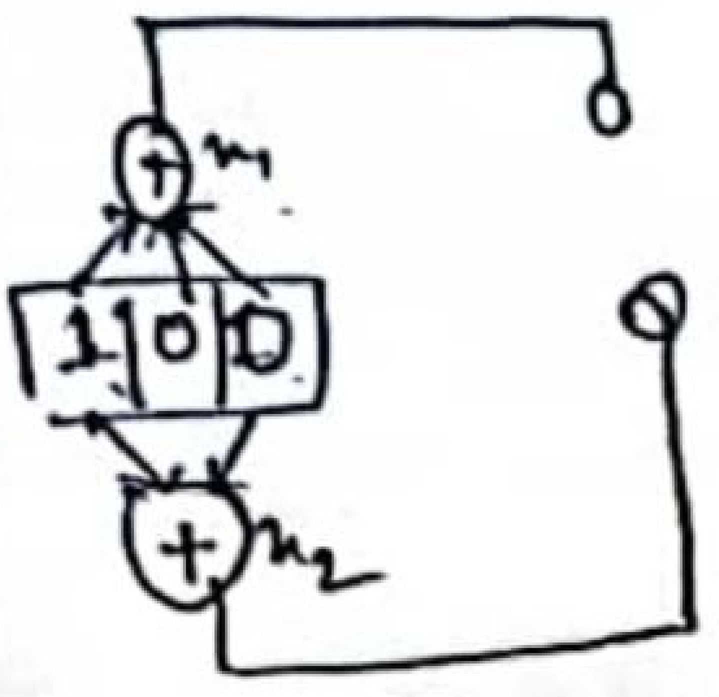
In convolution codes, block of 'n' code words generated by the encoder in time unit depends on not only block of 'K' message digits with in that time unit but also on the preceding (m-1) blocks of message bits



previous two successive message bit which represent state of convolution code

$m = 101$ → Here, Constant length = Number of register = 3

[এখানে নিয়ম হচ্ছে x_1 → যা উপরের ⊕ এর সাথে তিনটা বিটই add হবে। আর নিচের x_2 বা নিচের ⊕ " " First এবং Last bit add হবে]



$$x_1 = \cancel{m_1 \oplus m_2} \oplus m \oplus m_1 \oplus m_2$$
$$= 1 \oplus 0 \oplus 0$$
$$= 1$$

$$x_2 = m \oplus m_2$$
$$= 1 \oplus 0$$
$$= 1$$

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

⊛ properties of cyclic code:-

property 1:- property of Linearity

According to this property, a linear combination of two codewords must be another codeword.

Suppose, we have the codewords c_i & c_j.

so on adding,

$c_i + c_j = c_p$ [c_p = must also be a codeword]

Example:-

110	110	101
+	+	+
101	011	011
=	=	=
011	101	110

∴ $P(\alpha) = 0$

∴ Codeword polynomial equation,

$$C(x) = x^{n-k} M(x) + P(x)$$

$$= x^3(x^3+x+1) + 0$$

$$= x^6 + x^4 + x^3$$

∴ Code polynomial, $C = [1011000]$

(Ans)

☐ Systematic cyclic Encoding:-

~~M(x) =~~

Message signal : $M(x)$

Generator polynomial, $G(x)$

Systematic code, $C(x)$

parity polynomial, $P(x)$

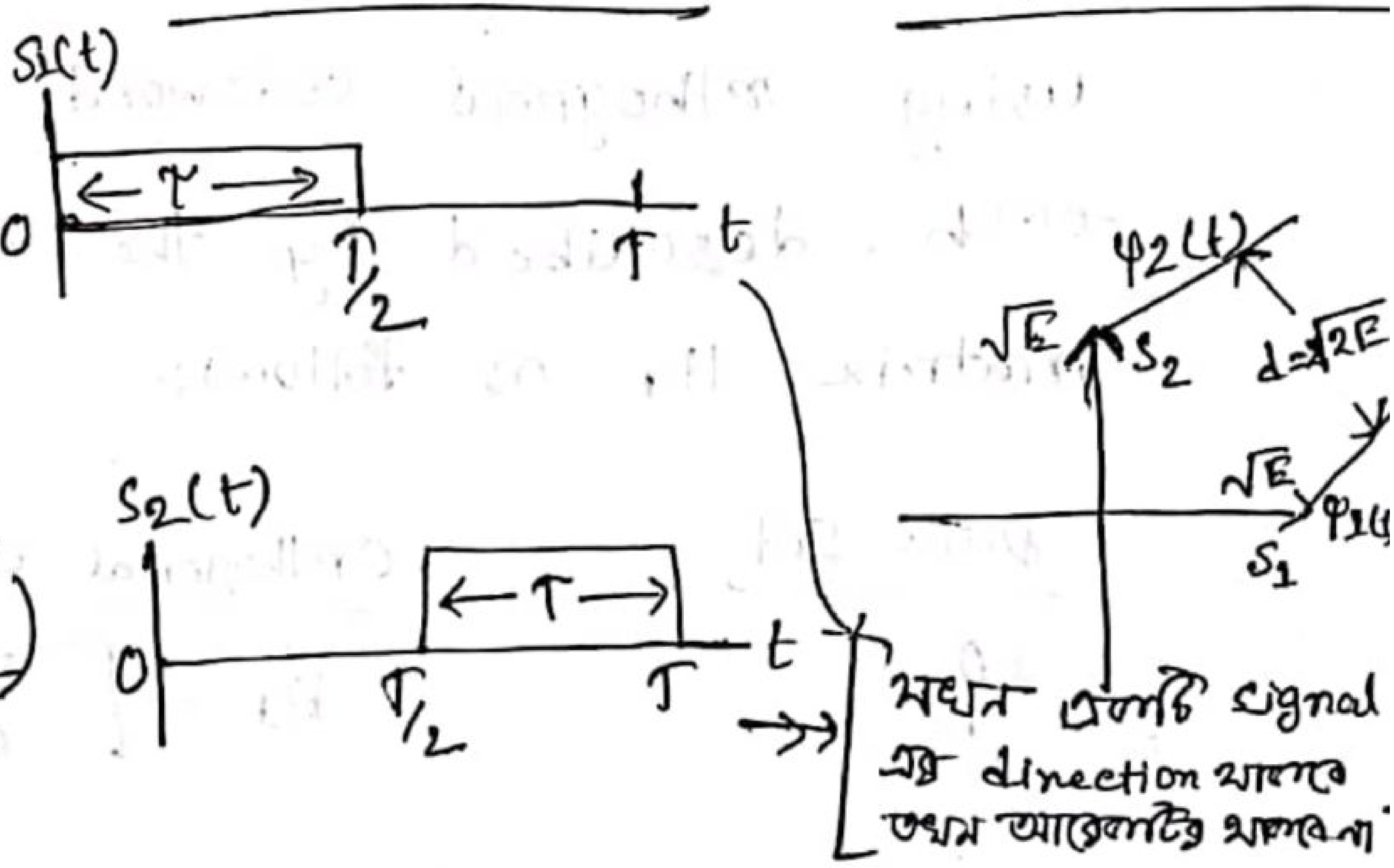
$$P(x) = \text{Remainder of} \left(\frac{x^{n-k} \cdot M(x)}{G(x)} \right)$$

Binary orthogonal signal set:-

Analytic representation | **waveform representation** | **vector representation**

$S_1(t) = p(t)$

$s_2(t) = p(t - T/2)$



যখন একটি signal এর direction থাকে তখন আরেকটির থাকে না

Figure:- Example of Binary orthogonal signal Set.

where, z_{ij} is called the cross-correlation coefficient, and where E is the signal energy, expressed as,

$$E = \int_0^T s_i^2(t)\, dt$$

Hamming code (Error Correction)

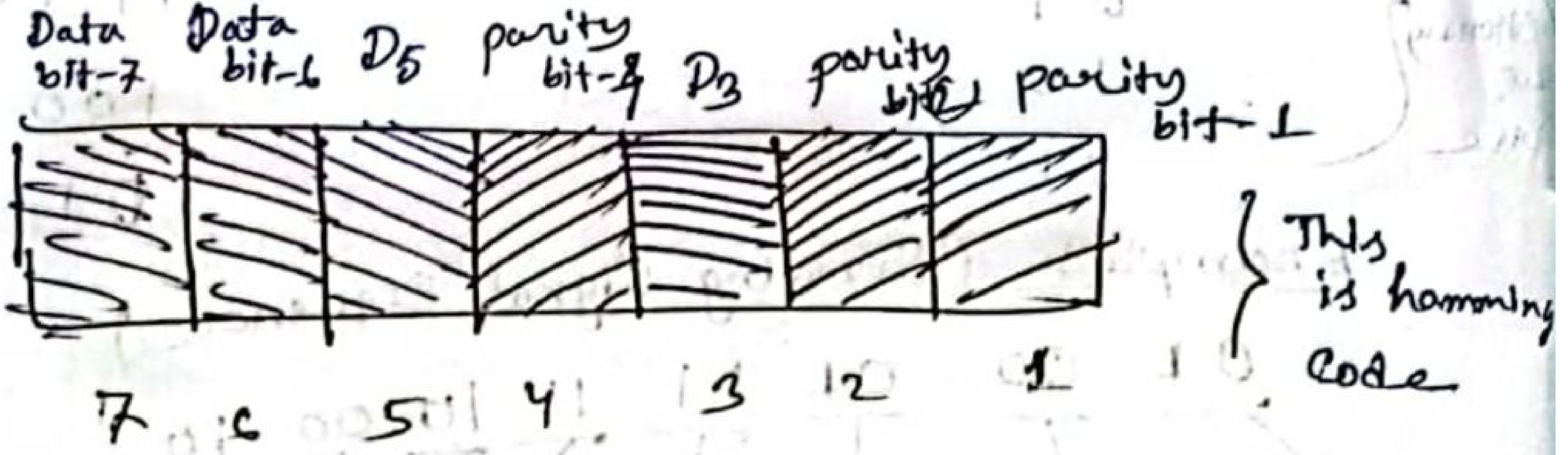
Data bits → 4 bit
parity bits → 3 bit
} For 7 bit hamming code.

Theory:-

2^n { where $n = 0, 1 \ldots n$ } will be position of parity bit.

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$

কত নং কোড hamming code হবে তা 2^n এর হিসাবে বলতে হবে।



1 0 1 0 → 1 is even parity here

$P_1 \rightarrow D_3\ D_5\ D_7$

$P_2 \rightarrow D_3\ D_6\ D_7$

$P_4 \rightarrow D_5\ D_6\ D_7$

} এগুলো মুখস্থ রাখতে হবে সূত্রের মতো

Linear Block Codes

Linear Block codes are a class of parity check code that can be characterized by (n, k).

The encoder transforms a block of k message digit (a message vector) into a longer block of 'n' codeword digits (a code vector) constructed from a given alphabet of elements. When the alphabet consists of two elements. The code is a binary code comprising binary digits (bits).

$$(n,k) = (\underbrace{5}_{n}, \overbrace{2}^{k})$$

Let's consider some block of data, which contains k bits in even block. These bits are mapped with the blocks which has n bits

$H^T =$

Syndrome Matrix:-

Receive vector r ⇒ [কোনো error থাকলে mixed, না থাকলে যেমন আছে তেমন]

$$r = u + e$$

$$r - e = u$$

↓

[X-OR operation হবে]

$UH^T = 0$ [T আসল কিনা তা check করে]

Syndrom,

$$S = rH^T$$

$$S = (u+e)H^T$$

$$= uH^T + eH^T$$

$$= 0 + eH^T$$

$$\therefore S = \underline{\underline{eH^T}}$$

$r = u + e$

(d)

Parity Check Matrix,

formula
$$G = \cancel{[I : P]}\ [P : I]$$
$$[P^T : I_{n-k}] = [P^T\ I_3]$$
$$H = \cancel{[P^T : I]}\ \cancel{[I : P^T]}$$

Here,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}}_{P^T} & \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \end{bmatrix}$$

Ex-02 (Transmitted codeword)

$$H = \begin{bmatrix} \overbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}^{I} & \underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}}_{I} \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

H_1 = 0 থাকলে 1 হবে
1 " 0 "

⊛ 3 bit Data set, $2^3 = 8$ data set,

Data Set

000
001
010
011
100
101
110
111

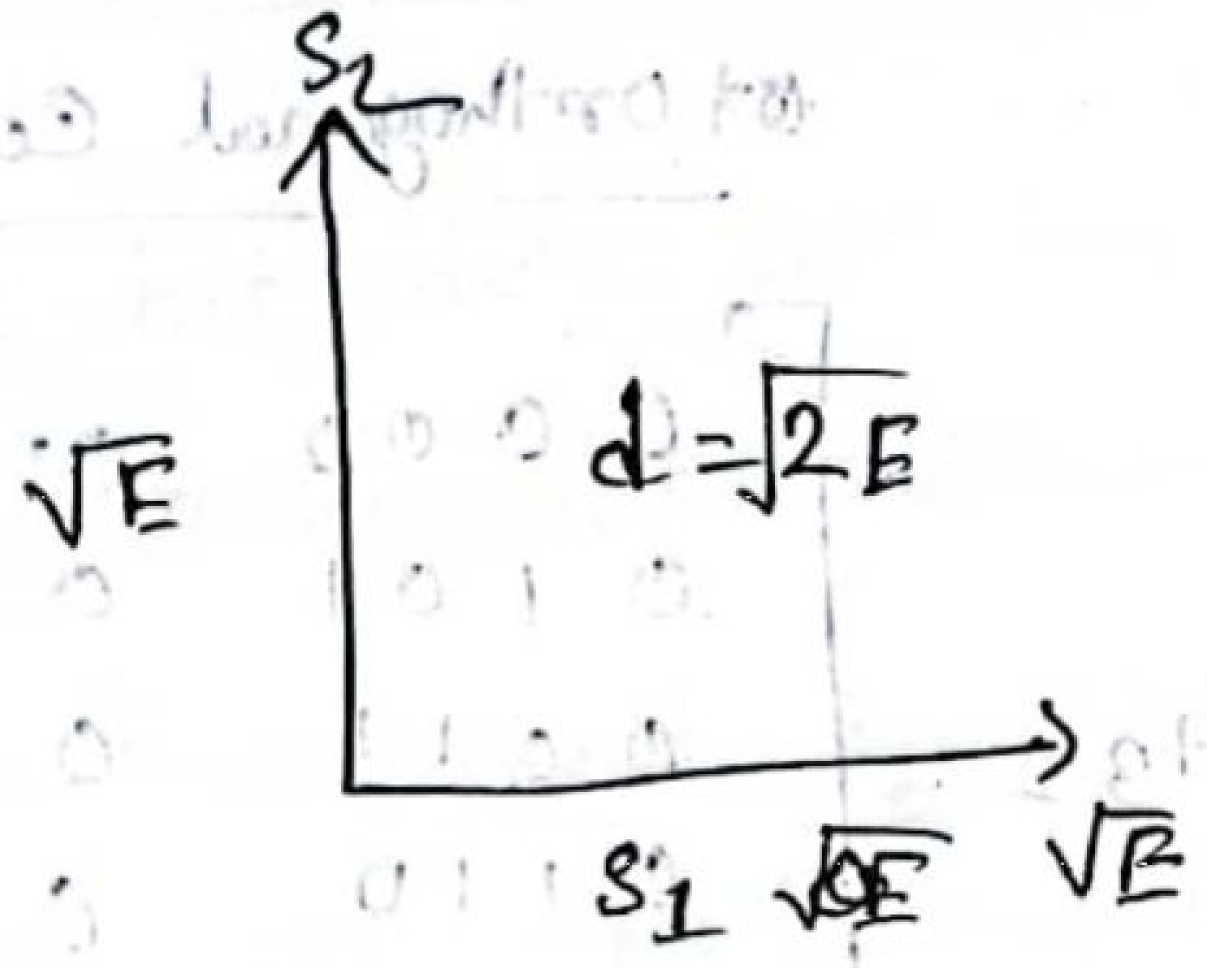
Orthogonal Codeword set

$$H_3 = \begin{bmatrix} 0000 & 0000 \\ 0101 & 0101 \\ 0011 & 0011 \\ 0110 & 0110 \\ 0000 & 1111 \\ 0101 & 1010 \\ 0011 & 1100 \\ 0110 & 1001 \end{bmatrix}$$

$$= \begin{bmatrix} H_2 & H_2 \\ H_2 & \overline{H_2} \end{bmatrix}$$

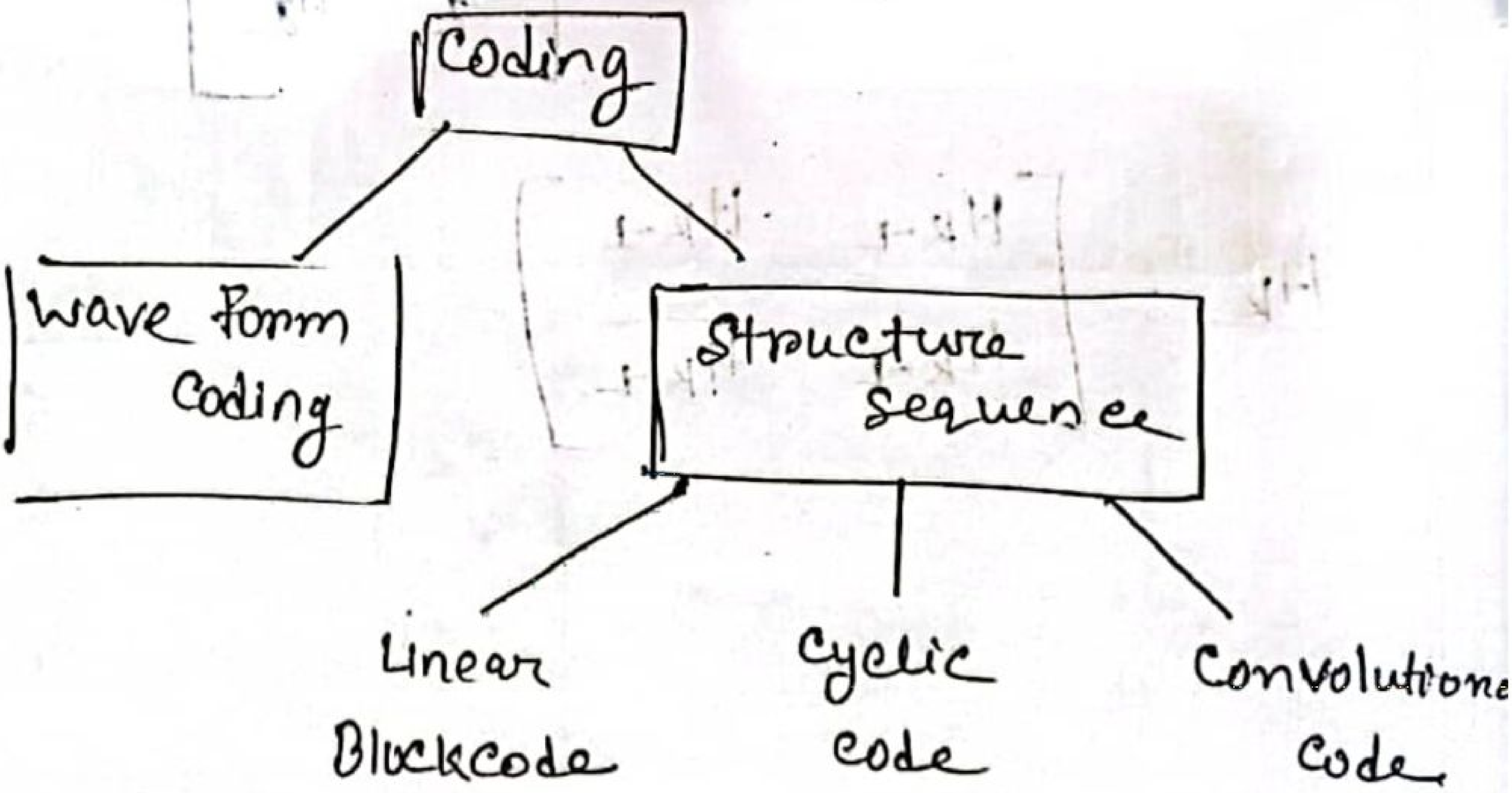
$$H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & \overline{H_{k-1}} \end{bmatrix}$$

Orthogonal Signal



VLSI

- We have to add additional bit with our message bit - Channel - coding.



Types of error control

Error correcting codes are classified according to their error correcting capabilities

Two types:-

→ ARQ (Automatic Repeat Request)

→ FEC (Forward Error Correction)

Automatic Repeat Request (ARQ)

① Tx ⟶ Rx

One direction
(Simplex)

② Tx ⟷ Rx

(Bidirectional)
(Half Duplex)

③ Tx ⇄ Rx

(Both directional)

$P_4 \ P_2 \ P_1 = (1 \ 0 \ 1)_2$

$= \cancel{(5)_{10}} (5)_{10}$

So the means, the 5th bit of the signal is having the error

which is $D_5 = 1$

As it is having error as 1 so it must be 0

So the exact signal will be

$\Rightarrow$ 1001011

ASCII

⊞ Channel coding:-

It can be partitioned into two study areas, waveform coding and structured sequence. (Add an extra bit with message bit to detect the error as well as correct the error)

Waveform coding

waveform coding techniques describe the waveform's instant behavior. This means that the waveform does not have to be speech in fact it can be analog data or a signaling tone.

⊛ Antipodal and orthogonal signals:-

An orthogonal signal set made up of pulse waveforms,

$$S_1(t) = P(t) \quad 0 \le t \le t$$

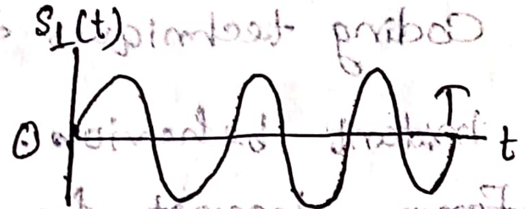
$$S_2(t) = P\left(t - \frac{T}{2}\right) \quad 0 \le t \le T$$

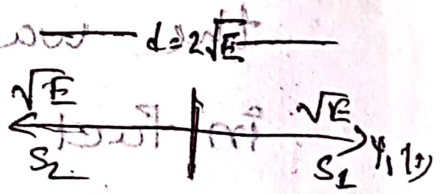
P(t) is a pulse, $T = T/2$ is the symbol duration.

Another orthogonal waveform set frequently used in communication system is sin x and cos x. $Z_{ij} = \frac{1}{E}\int_0^T s_i(T)\, s_j(t)\, dt = \begin{cases} 1 & \text{for } i=j \\ 0 \end{cases}$

Analytical representation — Waveform representation — Vector representation

Sig-1 $s_1(t) = \sin \omega_0 t$





Sig-2 $s_2(t) = -\sin \omega_0 t$

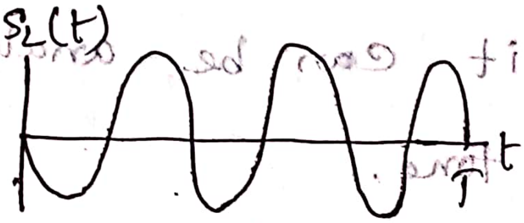


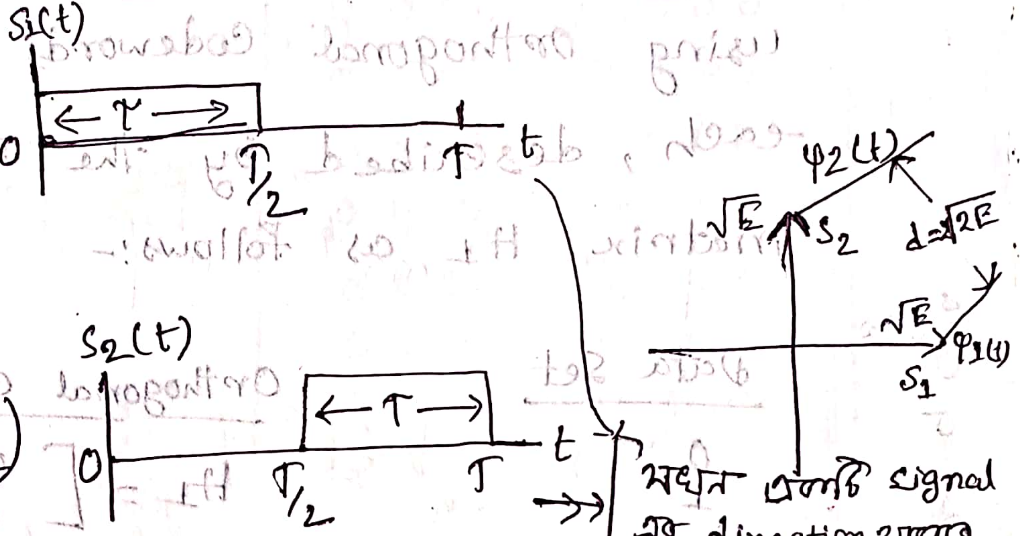
Figure:- Example of an antipodal signal set.

☐ Binary orthogonal signal set:

Analytic representation	waveform representation	vector representation

$S_1(t) = p(t)$

$S_2(t) = p\left(t - \frac{T}{2}\right)$



[যখন একটি signal এর direction থাকে তখন আরেকটির থাকবেনা]

Figure:- Example of Binary orthogonal signal Set.

where, zij is called the cross-correlation coefficient, and where E is the signal energy, expressed as,

$$E = \int_0^T s_i^2(t)\, dt$$

Orthogonal Code:-

- One bit data set can be transformed using orthogonal codewords of two digits each, described by the rows of matrix H_1 as follows:-

Data Set	Orthogonal codeword Set
0 1	$H_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

- To encode 2-bit data set, creating Matrix, H_2

Data Set	Orthogonal Codeword Set
0 0 0 1 1 0 1 1	$H_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} H_1 & H_1 \\ H_1 & \overline{H_1} \end{bmatrix} \Rightarrow$$

H = 0 থাকলে 1 হবে

1 ״ 0 ״

⊛ 3 bit Data set, $2^3 = 8$ data set,

Data set

000
001
010
011
100
101
110
111

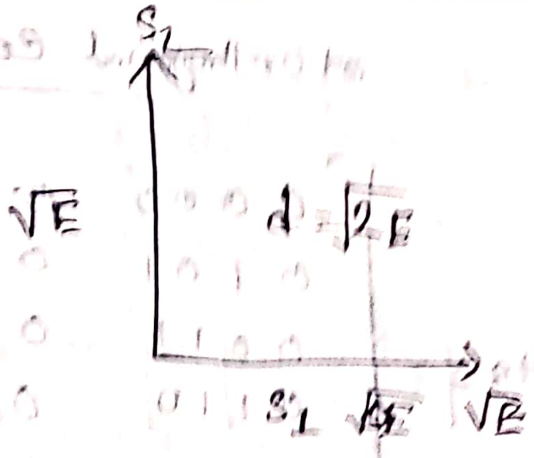
Orthogonal Codeword set

$$H_3 = \begin{bmatrix} 0&0&0&0&0&0&0&0 \\ 0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1 \\ 0&1&1&0&0&1&1&0 \\ 0&0&0&0&1&1&1&1 \\ 0&1&0&1&1&0&1&0 \\ 0&0&1&1&1&1&0&0 \\ 0&1&1&0&1&0&0&1 \end{bmatrix}$$

$$= \begin{bmatrix} H_2 & H_2 \\ H_2 & \overline{H_2} \end{bmatrix}$$

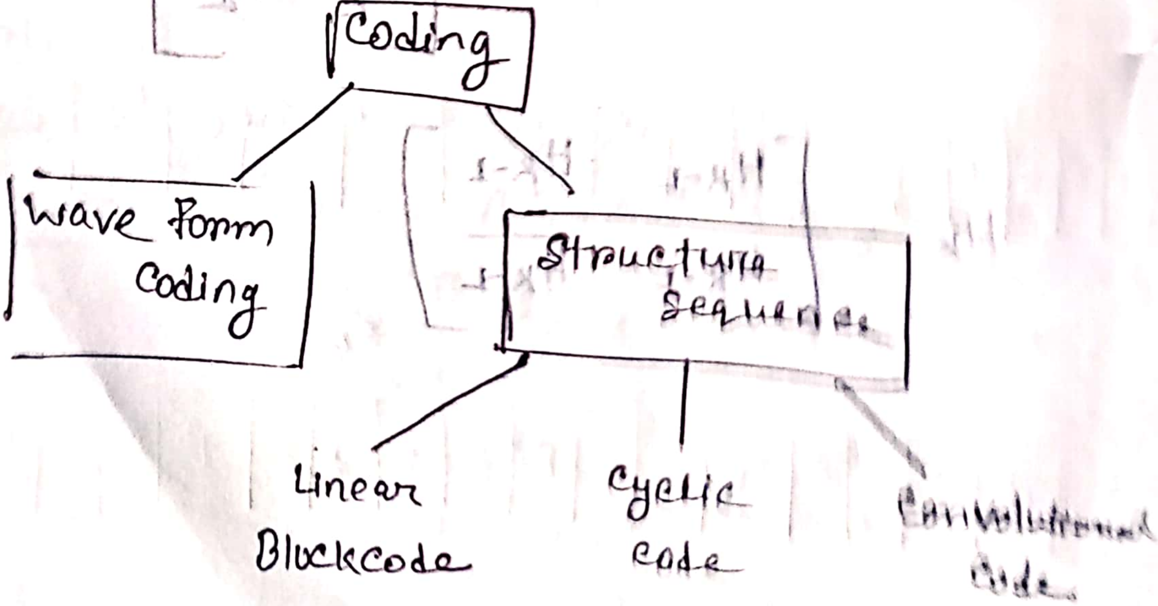
$$H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & \overline{H_{k-1}} \end{bmatrix}$$

Orthogonal Signal



VLSI

We have to add additional bit with our message bit - Channel - coding.



Types of error control

Error correcting codes are classified according to their error correcting capabilities

Two types:-

- → ARQ (Automatic Repeat Request)
- → FEC (Forward Error correction)

Automatic Repeat Request (ARQ)

① Tx ⟶ Rx
One direction
(Simplex)

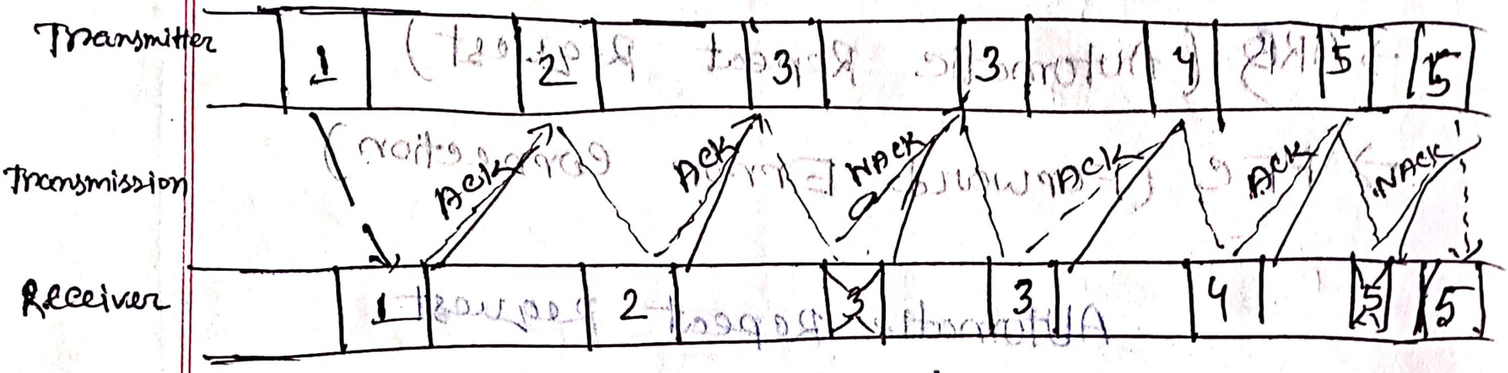
② Tx ⟷ Rx
(Bidirectional)
(Half Duplex)

③ Tx ⇄ Rx
(Both directional)

ARQ

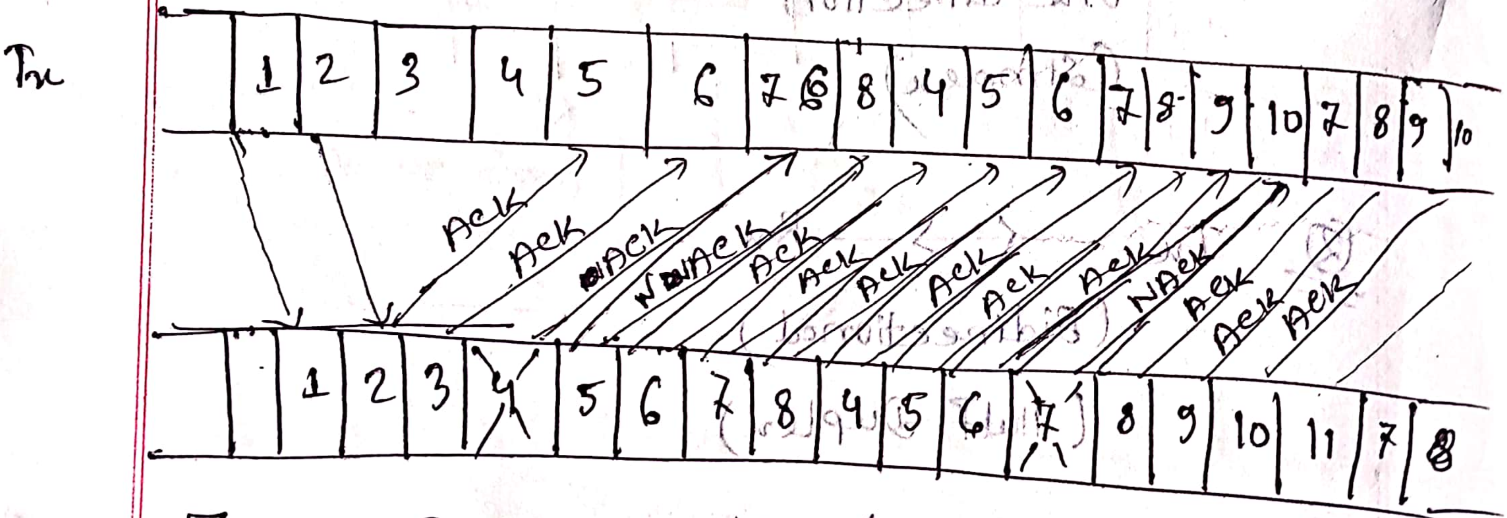
3 types:-

① STOP & wait:-



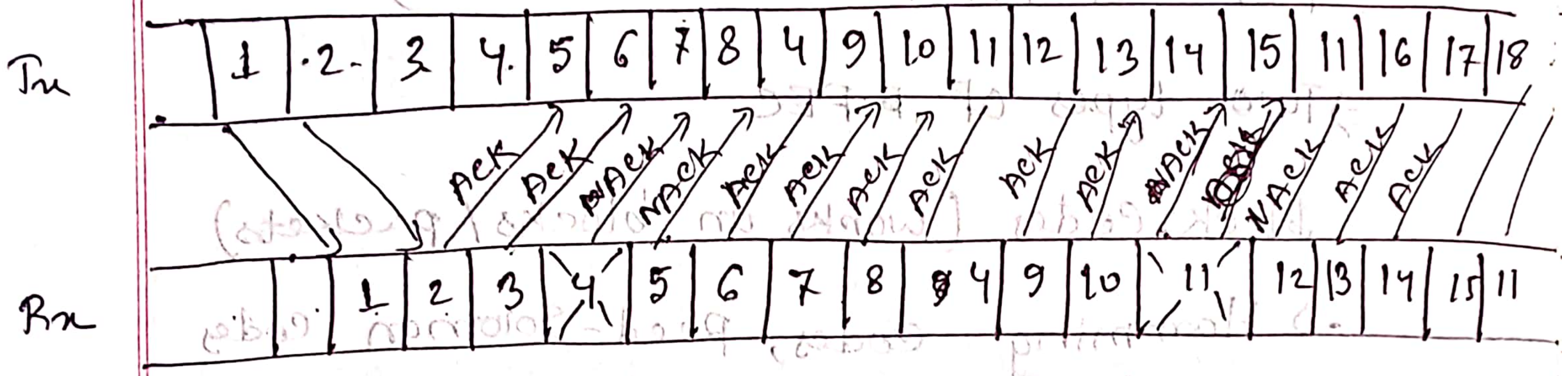
[এক্ষেত্রে ম্যাসেজ sending থেমে যাবে এবং সেখান হতে আবার সেখান হতেই স্টার্ট হবে)

② Go back N:-



[এক্ষেত্রে Transfer হতেই থাকবে তবে যদি কোনো error আসে তবে সেখান হতেই পুনরায় স্টার্ট হবে]

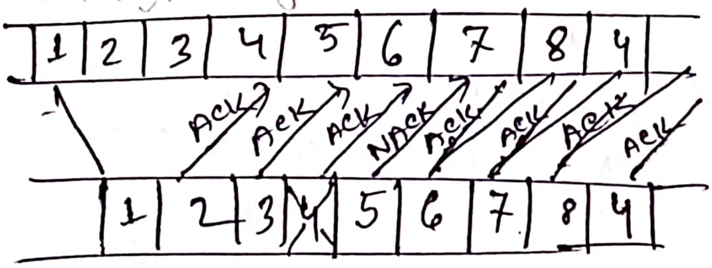
③ Selective & Repeat :-



[যেটা আসবে না সেখানে Not Acknowledgement signal পাঠাবে। এবং শুধুমাত্র ঐ জিনিসটাই পুনরায় পাঠাবে।]

Note:-

⊛ এখানে Ack NACK দুটো signal হতে পারে, অর্থাৎ,



FEC(Single channel):-

Forward Error Correction (FEC)

⇒ Two types of FEC

Block Codes (works on blocks / packets)

→ Hamming codes, Reed-Solomon codes

→ Convolutional codes (Arbitrary length symbols / bits)

uses:-

→ Reverse channel available না থাকলে

→ Retransmission covenient না হলে

→ Error is too much high হলে

Hamming Distance :-

101101
110100
× ×   ×

(যেসব value গুলোর Different

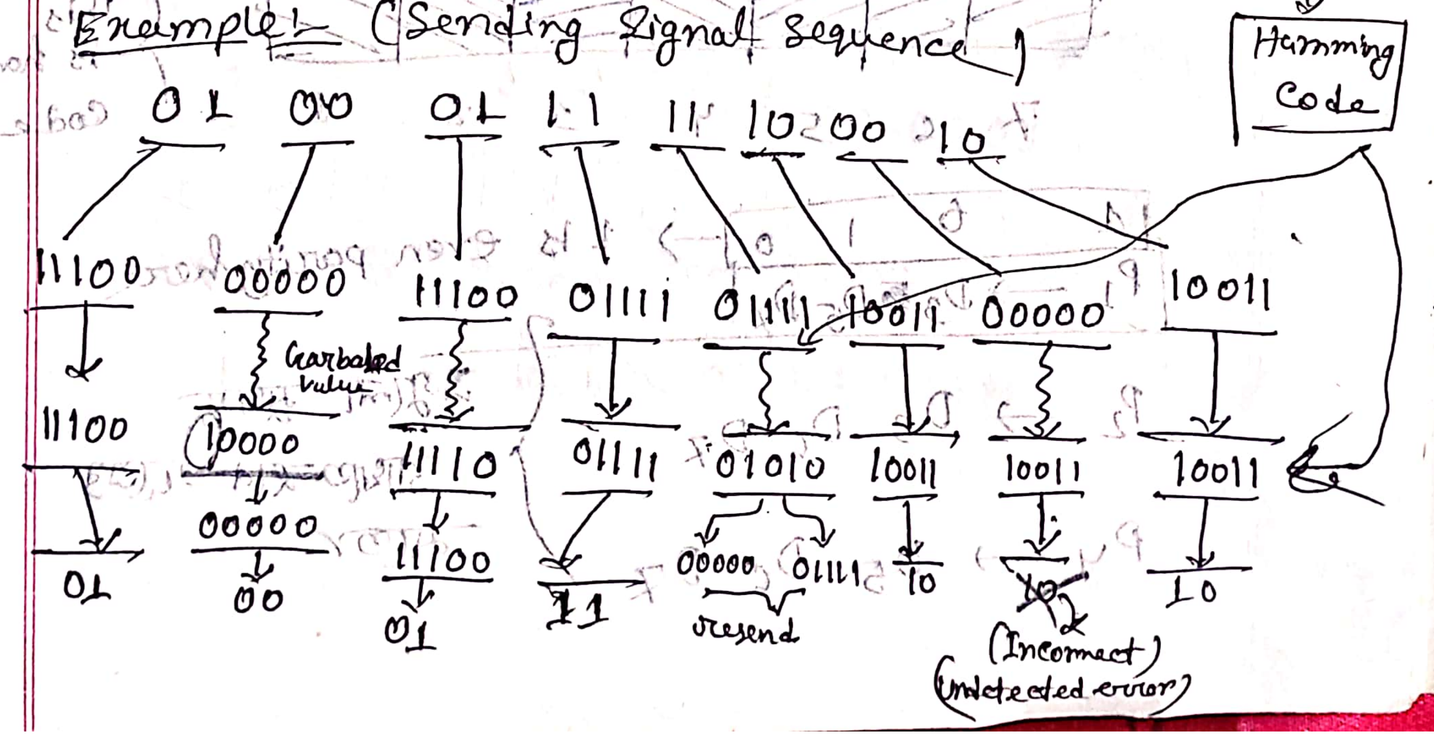
d(101101, 110100) = 3

Error correcting code (ECC)

The dictionary we have

encoded bit sequence	Corresponding codeword	
00	00000	1
01	11100	2
10	10011	2
11	01111	5

Example :- (Sending Signal Sequence)



Hamming code (Error Correction)

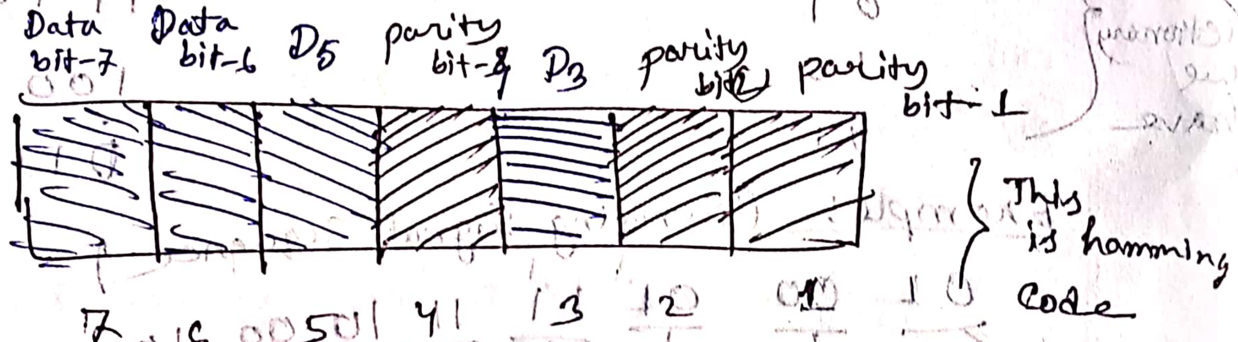
Data bits → 4 bit
parity bits → 3 bit } For 7 bit hamming code.

Theory:-

2^n { where $n = 0, 1 \ldots n$ } will be position of parity bit.

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$

কত নং কোড hamming code হবে তা 2^n এর হিসাবে করতে হবে।



1 0 1 0 → 1 is even parity here

$P_1 \rightarrow D_3 \; D_5 \; D_7$

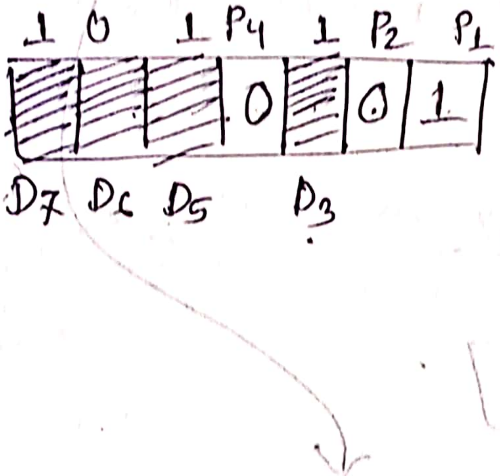
$P_2 \rightarrow D_3 \; D_6 \; D_7$

$P_4 \rightarrow D_5 \; D_6 \; D_7$

} এগুলো মুখস্থ রাখতে হবে সুত্রের মতো

Another example :-

1 0 1 1



P1 will depend on

→ D3 D5 D7

1 [1 1 1] so 1 is add here but we need to make it even so another 1 is needed so P1 = 1

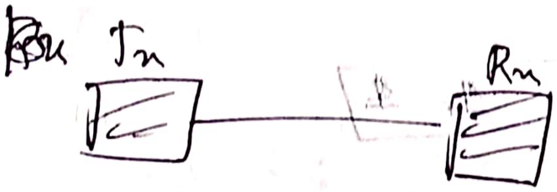
P2 = D3 D6 D7

0 [1 0 1]

P3 = D5 D6 D7

0 [1 0 1]

Transmitter & Receiver :-



we are going to send 1010101

The But there noise has been added. So the signal is changed to! 1110101

Solve :-

The receiver will see the parity bits first so,

$P_1 = D_3 \quad D_5 \quad D_7$

$= 1 \,|\, \boxed{1 \quad 1 \quad 1}$

$\therefore P_1 = 1$

1	1	1	0	1	0	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_2 = D_3 \quad D_6 \quad D_7$

$= 1 \,|\, \boxed{1 \quad 1 \quad 1}$

$P_2 = 1 \Rightarrow 1$ হবে কিন্তু error এর জন্য signal 0 হয়ে যাচ্ছে।

$P_4 = 1 \,|\,$ ~~$D_3\, 4\, D_5$~~ $D_5 \quad D_6 \quad D_7$

$= 1 \,|\, \boxed{1 \quad 1 \quad 1}$

$\therefore P_4 = 1 \Rightarrow$ কিন্তু error থাকায় signal 0 হয়ে আছে।

Hamming Code-Error Correction

Ex — If the 7-bit hamming code word received by a receiver is $\boxed{1011011}$. Assuming the even parity state whether the received code word is correct or wrong. If ~~wo~~ wrong locate the bit having error.

Solution:-

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	0	1	1	0	1	1

$P_4 = \boxed{1 \quad \overset{D_5}{1} \quad \overset{D_6}{0} \quad \overset{D_7}{1}} \rightarrow$ [odd আছে যা error]

$\rightarrow P_4 = 1$ (There is an error)

$P_2 = D_3 \quad D_6 \quad D_7$

$= 1 \quad 0 \quad 0 \quad 1 \rightarrow$ even (So there is no error

$P_2 = 0$ (As there is no error or contradiction)

$P_1 = D_3 \quad D_5 \quad D_7$

$= \boxed{1 \quad 0 \quad 1 \quad 1} \Rightarrow$ [3 -টা 1 আছে যা odd এবং error]

$P_4 \; P_2 \; P_1 = (1\,0\,1)_2$

$= (5)_{10}$

So the means, the 5th bit of the signal is having the error

which is $D_5 = 1$

As it is having error as 1 so it must be 0

So the exact signal will be,

$\Rightarrow 1001011$

Linear Block Codes

Linear Block Codes are a class of parity check code that can be characterized by (n, k).

The encoder transforms a block of k message digit (a message vector) into a longer block of 'n' codeword digits (a code vector) constructed from a given alphabet of elements. When the alphabet consists of two elements. The code is a binary code comprising binary digits (bits).

$$(n,k) = (\underset{n}{5}, \overset{k}{2})$$

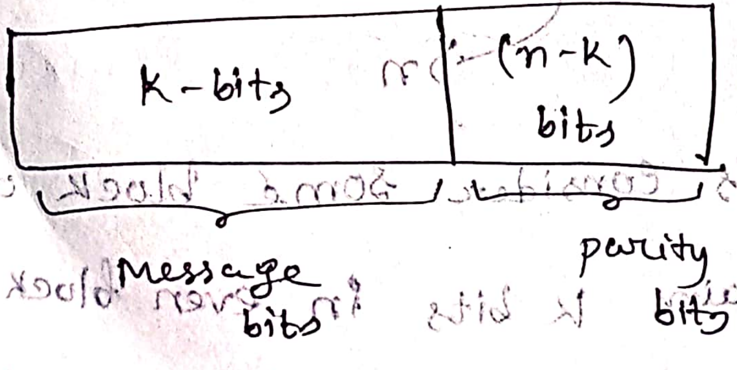
Let's consider some block of data, which contains k bits in even block. These bits are mapped with the blocks which has n bits in each block. Here, n is greater than k.

The transmitter adds redundant bits which are $n-k$ bits. The ratio k/n is the code rate. It is denoted by r and the value of r is $r<1$.

The $n-k$ bits are added here, are parity bits.

Any linear block code can be a systematic code, until it is altered. Hence, an unaltered block code is called as systematic code.

Structure of Codeword



Convolution Code

The total bits are send = n bits

message bits = k bits

∴ parity bits = $n-k$ bits

1100101 0101011 1010011 21

Example of blocks OF data

42 10011010100111 0001010010111000011101010001

Code rate :-

The redundancy is frequently expressed in terms OF the code rate.

The code rate R OF a code C OF size M and length n is,

$$R = \frac{\log_2 M}{n}$$

$$M = 2^k \quad , R = k/n$$

E201037
SAKUR

① The redundancy in ~~ifor~~ information Theory is the number of bits used to transmit a message minus the number of bits of actual information in the message.

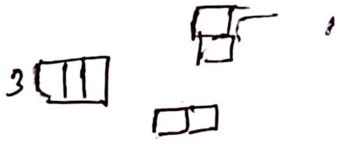
$$r = \cancel{n}\,n - \log_q M$$

Code word:- n bit encoded block of bits.

Block length:- The number of bits 'n' after coding is known as block length.

Code Rate:- Ratio of the number of message bits (k) to the total number of bits (n) in a code word.

$$r = k/n$$



Code vectors:-

An 'n' bit code word can be visualized in an n-dimensional space as a vector whose elements or coordinates are bits in the code word.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Hamming distance:-

It is the distance between the two codes expressed in the number of locations in which their respective elements differ.

Hamming weight of a code word [w(x)]:-

Defined as non-zero elements in the code word.

Code efficiency:-

The ratio of message bits to the number of transmitted bits per block. Code efficiency is equal to that of code rate.

~~Mimium~~ minimum distance (dmin):- It is defined as the smallest hamming distance between any pair of

Code vectors in the code.

Error:- when the output info doesn't match with input.

Error detecting code:-

Whenever a message is transmitted, it may get scrambled by noise or data may get corrupted. So to avoid this, we use error detecting codes.

(Here additional data added to a given digital message)

Example:- parity check.

Error Correcting Codes:-

We can pass some data to figure out the original message from the corrupt message that we received. This type of code is called as an error-correcting code.

⊛ Errors are detecting using parity check:-

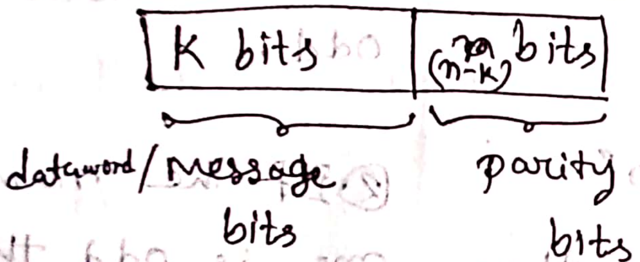
→ even parity	Odd parity
① The number of bits with a value of one are ~~connected~~ counted. If the value of 1 is odd we ~~make~~ add another 1 to make it even.	① If the number of bits with a value of one is an even number, the parity bit value is set to one to make the entire number of 1 as odd.
② If the value of even is even then we set the value of parity bit as 0.	② If the number of one is odd then we add zero to keep it odd.

~~Error Detection~~

~~Vector Space~~

Linear Block Coding:-

Message bits are divided into two blocks Each block contains K bits, and each K bits of a block defines a dataword.



(n,k)

The possible Codewords will be 2^n out of which 2^K contains datawords.

During transmission, if errors are introduced then this codewords will be changed.

In terms of Codewords and datawords, a term code rate is used,

$$r_c = \frac{K}{n}$$

Code rate:- Ratio of dataword bits to codeword bits.

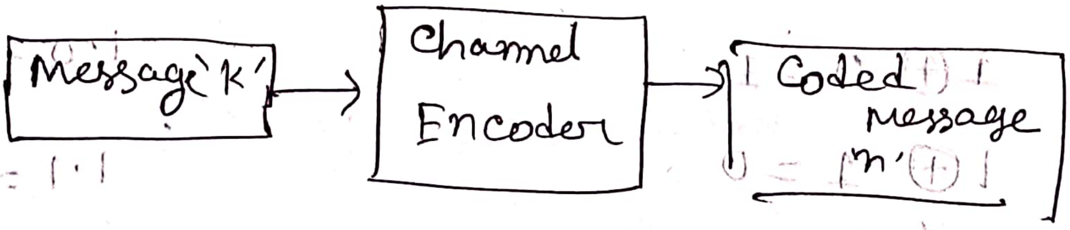
Linear Block codes: the data information is divided into ~~two~~ blocks of length k-bits data word.

$n = k + \overset{\text{parity bits.}}{p}$

vector notation is used for the Data word and code word.

→ For Data word, $m = (m_1, m_2, --, m_k)$

→ For Codeword, $u = (u_1, u_2, ----, u_n)$



There is always (one unique) codeword for each data word.

Code rate, $R = k/n$

Error Detection

⊞ Vector space :-

The set of all binary n-tuples V_n is called a vector space over the binary field of two element (0 & 1)

Binary field → Addition (X-OR)
Binary field → Multiplication

Addition (X-OR)	Multiplication
$0 \oplus 0 = 0$	$0 \cdot 0 = 0$
$0 \oplus 1 = 1$	$0 \cdot 1 = 0$
$1 \oplus 0 = 1$	$1 \cdot 0 = 0$
$1 \oplus 1 = 0$	$1 \cdot 1 = 1$

↳ (Modulo-2 operation)

⊞ vector Subspace:-

A subset S of the vector space, V_n is called a subspace of the following two conditions are met.

① All zeros vector is in S.

② The sum of any two vectors in S is also in S.

Suppose, V_i & V_j are two codewords (or Codevector) in an (n, k) binary block code. The code is said to be linear if and only if $(V_i \oplus V_j)$ is also a codevector.

[[Note!-

সাদা কথা হচ্ছে, এখানে যেকোনো দুইটি Codeword যোগ করলে যোগফলটি বাকি Codeword এর মধ্যেই থাকে]]

Examples:-

$V_4 \Rightarrow$ 4 tuples

$2^4 = 16$.

```
  0000
+ 0101
  ----
  0101
+ 1010
  ----
  1111
+ 1010
  ----
  0101
```

0000	0100	1000	1100
0001	0101	1001	1101
0010	0110	1010	1110
0011	0111	1011	1111

An example of a subset V_4 that forms a subspace is:-

0000 0101 1010 1111

A set of 2^K n-tuples is called a linear block code if and only if it is a subspace of the vector space V_n of all n-tuples.

Example:-

(6,3) Linear Block Code

→ K tuples Message vector

$2^K = 2^3 = 8$ message vector and therefore eight codeword.

There are $2^n = 2^6 = 64$ (6 tuples in the V6 vector space)

tuples = 6 का मतलब code word

$p = 6-3$

$P = n-k$

Message Vector	Codewords
000	000000
001	110 100
010	011 010
110	101 110
001	101 001
101	011 101
011	110 011
111	000 111

110 → parity code (Encoding)
100 → Message bit

Generator Matrix

$$\begin{array}{r} 011010 \\ \oplus\ 101001 \\ \hline 110011 \end{array}$$

$$G = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1\,1\,0\,1\,0\,0 \\ 0\,1\,1\,0\,1\,0 \\ 1\,0\,1\,0\,0\,1 \end{bmatrix}$$

v_1, v_2 & v_3 are three linearly independent vectors (a subset of 8 code vectors) that can generate all the code vectors.

(এগুলো এমন codeword হবে যেগুলো বাকি সব code vectors generate করতে পারবে)

$2^3 = 8$

Formula of Code Generate

$$U = m \times G$$

m = message vector

U = Code word

For ex!-

$$U_4 = [1\,1\,0]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= 1\cdot v_1 + 1\cdot v_2 + 0\cdot v_3 = v_1 + v_2$$

$= 110100 + 011010$

$= 101110$

$u_2 = [0\,0\,1] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

$= v_3$

$= 101001$

$u_3 = [0\,1\,0] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

$= v_2 = 011010$

- - - .

— —

(TO be continued)

(6, 3) n k

6, 3

2^n = Tuples সমান 2^k = Payloads.

2^k = vector /' [illegible] dimension হবে or

Systematic Linear Block code

A systematic (n, k) linear block code is a mapping from a k-dimensional message vector to an n-dimensional codeword in such a way that part of the sequence generated coincide with the k message digits. The remaining (n, k) digits are parity digits.

$$G = [P : I_k]$$

Given, message k-tuples

$$m = m_1, m_2, m_3, \ldots, m_k$$

general code vector n-tuples,

$$V = u_1, u_2 \ldots u_n$$

$2^k = 8 \rightarrow$ ☐

$\boxed{P = n - k}$

2^n

Systematic Codevector

$u = P_1, P_2, P_3 - - P_{n-k}$ $\qquad \underbrace{m_1, m_2, -- m_k}_{\text{Message bit}}$

$$u = [m_1, m_2, m_3] \begin{bmatrix} \underbrace{\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}}_{\substack{\cancel{P_{n-k}} \\ I}} & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\substack{I_k \\ P}} \end{bmatrix}$$

$$= \underbrace{m_1 \oplus m_3}_{u_1} \quad \underbrace{m_1 \oplus m_2}_{u_2} \quad \underbrace{m_2 \oplus m_3}_{u_3} \quad \underbrace{m_1}_{u_4} \quad \underbrace{m_2}_{u_5} \quad \underbrace{m_3}_{u_6}$$

$$\begin{bmatrix} \overset{m_1}{1} & \overset{m_2}{1} & \overset{m_3}{1} \end{bmatrix} = 0\ 0\ 0\ 1\ 1\ 1$$

$1 \oplus 1 = 0$

$$[1\ 0\ 1] = 0\ 1\ 1\ 1\ 0\ 1$$

VVIm

PAAni

$n \to$ code word
$k \to$ Tuples $\to$ messages
$2^k =$ message sequences
$= 2^3 = 8$

Question-

For a $\overset{n\ \ k}{(6, 3)}$ code, the generator matrix

G is,

$$G = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} & \underbrace{\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}}_{P} \end{bmatrix}$$

I:P

Find,

(a) All corresponding code vectors.

(b) Minimum Humming Distance

(c) verify that this code is a single-error correcting code.

(d) parity check Matrix

(e) Determine Transmitted codeword if received codeword is : 100011

(a)

$$G = \begin{bmatrix} \underbrace{100}_{I} & \overset{\xrightarrow{\text{P or parity bit}}}{101} \\ 010 & 011 \\ 001 & \underbrace{110}_{P} \end{bmatrix}$$

6,3

Finding all corresponding code vectors

we know,

$$G = [\overset{3\text{ bit}}{I} : \overset{3\text{ bit}}{P}]$$

~~G = [P I]~~

[C] = [m] [P]

Here

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Code bits, $C = n - k$

$= 6 - 3 = 3$ bit

Message bit, $M = 2^k = 2^3 = 8$ bit

Now, $[C] = [m]\,[P]$

$$[C_0, C_1\ C_2] = [M_0 M_1 M_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$C_0 = M_0 \oplus M_2$, $C_1 = M_1 \oplus M_2$, $C_2 = M_0 \oplus M_1$

$C_0 = (M_0 \oplus M_2)$, $C_1 = (M_1 \oplus M_2)$, $C_2 = (M_0 \oplus M_1)$

M_0	M_1	M_2	C_0	C_1	C_2
0	0	0	0	0	0
0	0	1	1	1	0
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	0	0

(b)

∴ Minimum Humming distance,

M_0	M_1	M_2	C_0	C_1	C_2	Minimum H.D
0	0	0	0	0	0	→ 0
0	0	1	1	1	0	→ 3 ✓
0	1	0	0	1	1	→ 3
0	1	1	1	0	1	→ 4
1	0	0	1	0	1	→ 3
1	0	1	0	1	1	→ 4
1	1	0	1	1	0	→ 4
1	1	1	0	0	0	→ 3

(কয়টা one আছে সেই সংখ্যাটাই Humming distance)

$\therefore$ Minimum Humming distance

$d_{min} = 3$

(Ans)

②

This code is single bit error ~~coding~~, Correcting code,

① $d_{min} \geq s+1$ [Here, s = No. of error detection]

$\therefore 3\cancel{6} \geq s+1$

$\Rightarrow 2 \geq s$

or $s \leq 2$

That means, this code detect 2 bit error in 6 bit message

(ii) $d_{min} \geq 2t+1$ [Here, t = No. of error correction]

$3 \geq 2t+1$

$\therefore t \leq 1$

$t \leq 1$ means that 2 bit error detected message ~~coppe~~corrected a 1 bit.

(d) Parity check Matrix:

Theory:—

Parity Generator, $G = [P : I_k]$ $[I_k : P]$

(n,k) , $P = n-k$

Parity Check Matrix:—

→ Transpose

~~$H = [I_{n-k} : P^T]$~~

$H = [P^T : I_{n-k}]$

$H^T =$ ~~[illegible]~~

Syndrome Matrix:-

Receive vector r ⇒ [কোনো error থাকলে mixed, না থাকলে যেমন আছে তেমন]

$r = u + e$

$r - e = u$

[X-OR operation হবে]

$uH^T = 0$ [T আসল কিনা তা check করে]

Syndrom,

$S = rH^T$

$S = (u+e)H^T$

$= uH^T + eH^T$

$= 0 + eH^T$

$\therefore S = \underline{eH^T}$

$r = u + e$

$G = [P \quad I]$

$H = [I_{n-k} \quad P^T]$

ⓓ

Parity Check Matrix,

formula

$G = \cancel{[I : P]} \; [P : I]$

$H = \cancel{[P^T : I]} \; \cancel{[I : P^T]} \; [P^T : I_{n-k}] = [P^T \; I_3]$

Here,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1\,0\,1 & 1\,0\,0 \\ 0\,1\,1 & 0\,1\,0 \\ 1\,1\,0 & 0\,0\,1 \end{bmatrix}$$

P^T

Ex-02 (Transmitted codeword)

$$H = \begin{bmatrix} \overbrace{1\,0\,0}^{I} & 1\,0\,1 \\ 0\,1\,0 & 1\,1\,0 \\ 0\,0\,1 & \underbrace{0\,1\,1}_{I} \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$2^{r} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = m$$

(e) Determining Transmitted codeword, if received codeword is 100011

we know,

$$S = \overset{\text{received codeword}}{r} \cdot [H^T]$$

$1 \oplus$

$$= [100011] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [(1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0) \ (0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0) \ (1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1)]$$

$$= [1 \quad 1 \quad 0]$$

∴ Transmitted code word = 101011

(Any)

Syndrome look up Table

Error pattern (e)	Syndrome
000000	000
000001	101
000010	011
000100	110

Decoder

(6,3) [illegible] का

$S = r H^T$

$$= \begin{bmatrix} r_1, r_2, r_3, r_4, r_5, r_6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$s_1 = r_1 + r_4 + r_6$

$s_2 = r_2 + r_4 + r_5$

$s_3 = r_3 + r_5 + r_6$

110

$$\begin{array}{r} 100011 \\ \oplus\,000110 \\ \hline 1000101 \end{array}$$

Syndrome Testing

~~Sap.~~

Problem:-

Suppose a received vector $r = 001110$ is received and find the syndrome vector value, $S = r \cdot H^T$

And verify that is equal to eH^T

Where,

$$G = \begin{bmatrix} \overbrace{1\ 1\ 0}^{P} & \overbrace{1\ 0\ 0}^{3=R} \\ 0\ 1\ 1 & 0\ 1\ 0 \\ 1\ 0\ 1 & 0\ 0\ 1 \end{bmatrix}$$

Solution:-

$S = rH^T$

011
110

$G = [P\ P']$
$H = [I\ P^T]$

$$G = \begin{bmatrix} 110 & 100 \\ 011 & 010 \\ 101 & 001 \end{bmatrix}$$

$$S = rH^T = \begin{bmatrix} 001110 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} I_{n-k} & \vdots & P^T \end{bmatrix}$$

$$= \begin{bmatrix} I_m & P^T \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{1\ 00\ 101} \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 100 \end{bmatrix}}$$

~~$S = e \cdot H^T$~~

$$H = \begin{bmatrix} I_3 & P^T \end{bmatrix} = \begin{bmatrix} 100 & 1\ 0\ 1 \\ 010 & 1\ 1\ 0 \\ 001 & 0\ 1\ 1 \end{bmatrix}$$

$$\begin{array}{r} 001110 \\ -000100 \\ \hline 001010 \end{array}$$

$\therefore S = r H^T$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \oplus 1 \oplus, & 1 \oplus 1, & 1 \oplus 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

vector r.

From error pattern we get,

Syndrome — **Error pattern**

100 — 100000

[illegible], $r = u + e$

$u = r + e$

$= 001110 + 100000$

$= 101110$

Syndrome look up Table

Error pattern	Syndrome
000 000 0	000
000 001	101
000 010	011
000 100	110
— —	— —

(6,3) decoder,

$$= \left[r_1, r_2, r_3, r_4, r_5, r_6\right] \begin{bmatrix} 100 \\ 010 \\ 001 \\ 110 \\ 011 \\ 101 \end{bmatrix}$$

$S_1 = r_1 + r_4 + r_6$

$S_2 = r_2 + r_4 + r_5$

$S_3 = r_3 + r_5 + r_6$

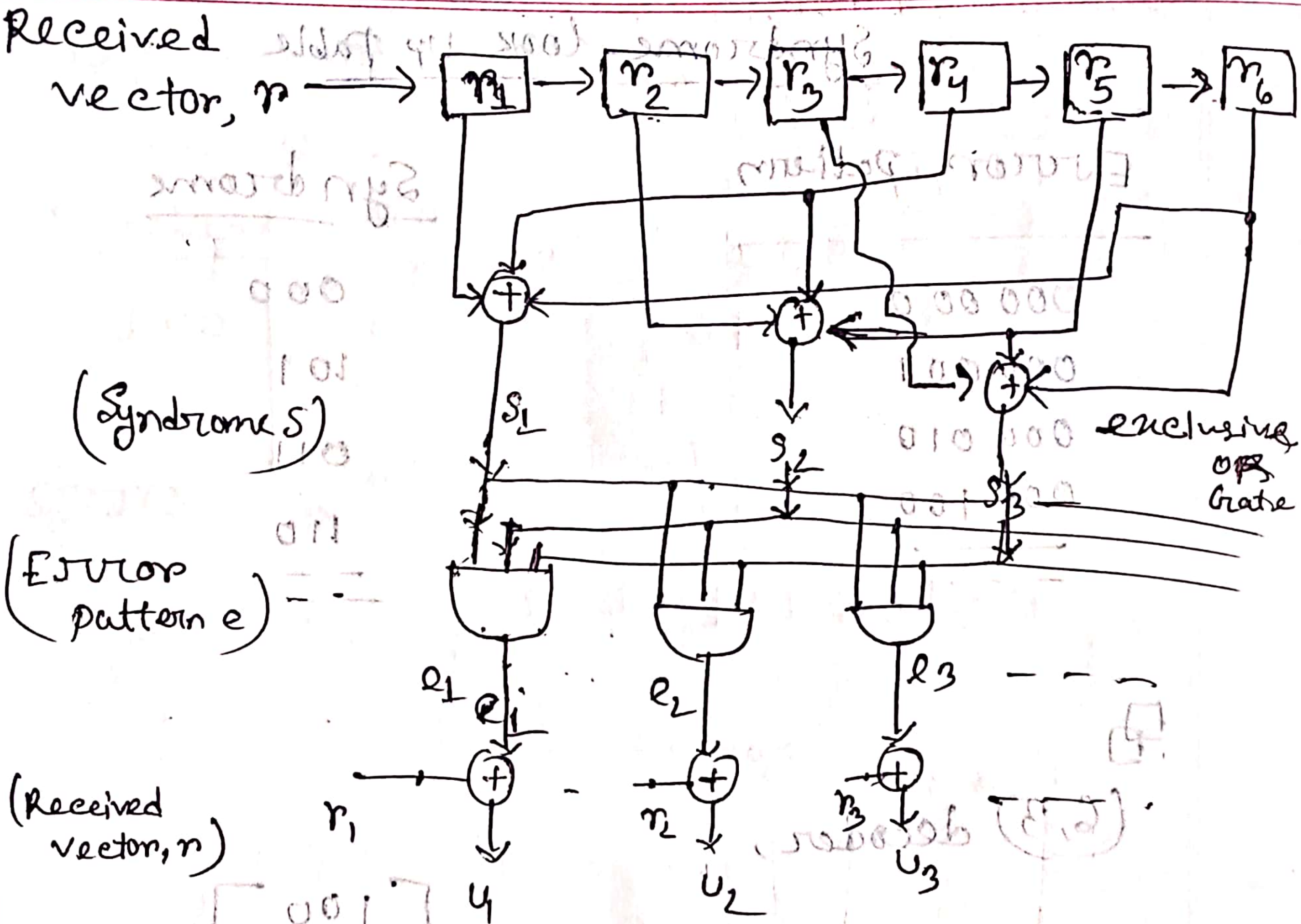


Fig:- Implementation of the (6,3) decoder.

e_1	e_2	e_3	e_4	e_5	e_6	S_1	S_2	S_3
0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1
0	0	0	0	1	0	0	1	1
0	0	0	1	0	0	1	1	0
0	0	1	0	0	0	0	0	1
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—

$e_1 = S_1 \bar{S}_2 \bar{S}_3$

$G = [P \ I_k]$

$H = [I_{n-k} \ P^T]$

Class Test question

[P] Consider a (5,2) linear Systematic block code defined by the generator matrix,

$$G = \begin{bmatrix} g_0 \\ g_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

P

(a) Find all the code vector ~~words~~ of the above block code.

(b) Find the parity check Matrix for this code

(c) How many errors can the code correct

(d) Find all the code words of the above block code

(Ans) (a)

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

~~P~~ I ~~E~~ P

Here,

The linear system is $(5, \overset{k}{2})$

Here,

message bit = 2 ~~bit~~ tuples

$G = \begin{bmatrix} P_k & I \end{bmatrix} \quad \begin{bmatrix} M \\ 10 \end{bmatrix}$.

$: \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

And $2^2 = 4$ bit. $p^T : \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

parity $= 6 \cdot 5 - 2 = 3$ bit

we know,

$[C] = [m] \quad [P]$

So,

Code $= 5 - 2 = 3$ bit

$$\begin{bmatrix} c_0 & c_1 & c_2 \end{bmatrix} = \begin{bmatrix} m_0 & m_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} m_0 & m_1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$= \cdot\, m_1 \quad m_0 \quad m_1$

$c_0 = m_1$

$c_1 = m_0$

$c_2 = m_1$

As the message bit is 4bit

~~0 0~~

~~0 1~~

m_0	m_1	c_0	c_1	c_2
0	0	0	0	0
0	1	1	0	1
1	0	0	1	0
1	1	1	1	1

(Ans)

(b)

Here,

$$G = \begin{bmatrix} I_K & P \end{bmatrix} = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix}}_{I} & \underbrace{\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}}_{3P} \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\therefore P^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

∴ Parity Matrix,

$$H = \cancel{\begin{bmatrix} I & P^T \end{bmatrix}} \begin{bmatrix} P^T & I_{n-K} \end{bmatrix} = \begin{bmatrix} P^T & I_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ & \cancel{0}\,1 & & \end{bmatrix}$$

(c)

Here

$$\begin{array}{ccccc|c} m_0 & m_1 & c_0 & c_1 & c_2 & H/D \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 5 \end{array}$$

∴ Minimum Humming distance = 2

NO. OF error detection:-

$d_{min} \geq s+1$

or, $2 \geq s+1$

$\Rightarrow 2-1 \geq s$

$\Rightarrow 1 \geq s$

∴ This code detect 1 bit error in 5 bit message

NO. OF error correction,

$d_{min} \geq 2t+1$

$\Rightarrow 2 \geq 2t+1$

$\Rightarrow 1 \geq 2t$

$\Rightarrow t \leq \frac{1}{2}$

∴ It does $\frac{1}{2}$ bit error correction.

(d) Here $(n,k) = (5,2) = 2 = K$

$U = K \cdot G$

$\therefore$ Total message bit $= 2^K = 2^2 = 4$

k
0 0
0 1
1 0
1 1

Total codeword $= 4$

$\therefore$ we know,

$U = K \cdot G$

$$U_1 = [00] \cdot G$$

$$= [00] \begin{bmatrix} 1 1 0 1 0 \\ 1 0 1 0 1 \end{bmatrix}$$

$$= 00000$$

$$U_2 = [01] \begin{bmatrix} 1 1 0 1 0 \\ 1 0 1 0 1 \end{bmatrix}$$

$$= [0 \oplus 1 \quad 0 \oplus 0 \quad 0 \oplus 1 \quad 0 \oplus 0 \quad 0 \oplus 1]$$

$$= [1 \quad 0 \quad 1 \quad 0 \quad 1]$$

$U_3 = [1\ 0] \begin{bmatrix} 1\,1\,0\,1\,0 \\ 1\,0\,1\,0\,1 \end{bmatrix}$

$= [1\oplus 0 \quad 1\oplus 0 \quad 0\oplus 0 \quad 1\oplus 0 \quad 0\oplus 0]$

$= [1 \quad 1 \quad 0 \quad 1 \quad 0]$

$u_4 = [1\ 1] \begin{bmatrix} 1\,1\,0\,1\,0 \\ 1\,0\,1\,0\,1 \end{bmatrix}$

$= [1\oplus 1 \quad 1\oplus 0 \quad 0\oplus 1 \quad 1\oplus 0 \quad 0\oplus 1]$

$= [\cancel{1}\ 0 \quad 1 \quad 1 \quad 1 \quad 1]$

∴ Code word

$$= \begin{bmatrix} 0\,0\,0\,0\,0 \\ 1\,0\,1\,0\,1 \\ 1\,1\,0\,1\,0 \\ 0\,1\,1\,1\,1 \end{bmatrix}$$

(Ans)

⊞ Error Correcting

$$t = \left[\frac{d_{min} - 1}{2} \right]$$

$$= \frac{3-1}{2}$$

$$= \frac{2}{2} = \underline{1}$$

(Ans)

Minimum Haming distance

00000 – 0
10101 – ③
11010 – 3
01111 – 4

Hamming weight

Hamming weight $w(U)$ of a codeword U is defined to be the number of non-zero elements in U.

=> $U = 100101101$

$w(U) = 5$

$V = 011110100$

⊞ Hamming Distance :-

$$\begin{array}{rl} & U = 100101101 \\ & V = 011110100 \\ \oplus & \\ \hline & 111011001 \end{array}$$

d(u,v) pair এর মধ্যে যখন same হবে না তখন Hamming Distance

=> Hamming Distance = 6

$$\Rightarrow d(u,v) = \overset{\text{Hamming weight}}{w}(u+v)$$

That means, The Hamming distance is equal to the Hamming weight of the new codeword.

Cyclic Shift of a codevector

⊛ Let, U = 1101 for $n=4$

Express the codeword in polynomial form. Solves for the third end around shift of the codeword

Theory:

Suppose, we have a bit = 101 => 5 in decimal

So, Bit to Decimal Deci

$1\times2^2+1\times2^1+1\times2^0=5$

Here, we will use x in the place of 2^n,

Solution:->

U = 1101

$U(x) = 1\times x^0 + 1\times x^1 + 0\times x^2 + 1\times x^3$

$= 1 + x + x^3$

NOW, i = Number of shifting

$$x^i \cdot U(x) = x^3(1+x+x^3)$$

$$= x^3+x^4+x^6 \quad [\text{where } i=3]$$

[Note :- কোনো Codeword কে Generator Matrix দিয়ে divide করলে New codeword পাওয়া যায়]

Divide, $x^i\,U(x)$ log x^n $= G = x^4+1$

$$
\begin{array}{r l}
x^4+1\,\big) & x^3+x^4+x^6\ (1+x^2 \\
 & x^4 \qquad +1 \\
\hline
 & x^3+x^6+1 \\
 & \cancel{x^3}\,x^6+x^2 \\
\hline
 & x^3+1+x^2
\end{array}
$$

∴ Reminder : $1+x^2+x^3$ [illegible]

So, from polynomial to Bit

$U^3 = 0\ 1\ 0\ 1\ 1$

properties of cyclic code

Cyclic code : Subclass of a linear block codes where cyclic shift in the bits of the codeword results in another codeword. (অর্থাৎ, linear Block code এ যদি cyclic code use করে আরেকটি Code তৈরি করা হয় তবে সেই Codeword ও অন্য যেকোনো Codeword এ পাওয়া যাবে।

=) It is widely used in satellite communication as the information sent digitally encoded & decoded using cyclic coding. ~~These are~~

These are error correcting codes where the actual information is sent over the channel by combining with the parity bits. Various other important codes like, Reed Solomon, Golay, Hamming, BCH, etc can be represented using cyclic codes.

⊛ properties of cyclic code:-

property 1:- property of Linearity

According to this property, a linear combination of two codewords must be another codeword.

Suppose, we have the codewords c_i & c_j.

so on adding,

$c_i + c_j = c_p$ [c_p = must also be a codeword]

Example:-

$$\begin{array}{ccc} 110 & 110 & 101 \\ + & + & + \\ 101 & 011 & 011 \\ = & = & = \\ 011 & 101 & 110 \end{array}$$

property ~~of~~ 02 :- property of cyclic shifting.

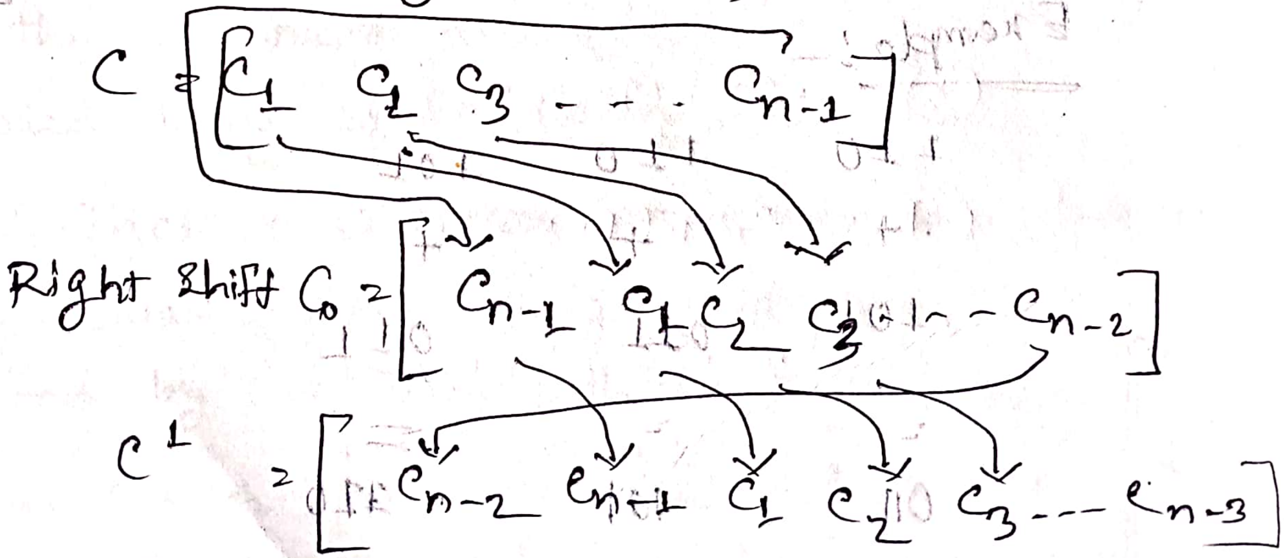
After a left or right shift in the bits of codewords the resultant code generated must be another codeword.

Suppose,

C is a codeword given as:

$$C = [c_1, c_2, \ldots c_{n-1}]$$

Then after cyclic shifts,



$$C = [c_1 \; c_2 \; c_3 \ldots c_{n-1}]$$

Right shift $C_0 = [c_{n-1} \; c_1 \; c_2 \; c_3 \ldots c_{n-2}]$

$$C^1 = [c_{n-2} \; c_{n-1} \; c_1 \; c_2 \; c_3 \ldots c_{n-3}]$$

Example:-

110 : right shift will provide: 011

101 : " " " " 110

011 : " " " " 101

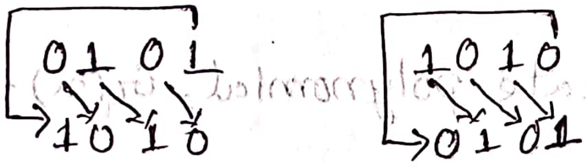
⊛ Consider code {0000, 0101, 1010, 1111} is it a cyclic code?

=)

$$\begin{array}{r} 0101 \\ \oplus\ 1010 \\ \hline 1111 \end{array} \qquad \begin{array}{r} 0101 \\ \oplus\ 1111 \\ \hline 1010 \end{array} \qquad \begin{array}{r} 1010 \\ \oplus\ 1111 \\ \hline 0101 \end{array}$$

It follows the property of linearity.

Now, check the property of shifting:-



It follows the property of shifting.

So, given code is cyclic code

Encoding

Non systematic cyclic Encoding :-

Consider message signal given as,

$m = [1110]$

$M(x) = 1 \times x^3 + 1 \times x^2 + 1 \times x^1 + 0 \times x^0$

$= x^3 + x^2 + x$

Here, Generate polynomial, $G(x) = x^3 + x + 1$

Non-systematic code, codeword,

$$C(x) = M(x) \cdot G(x)$$

$\therefore C(x) = (x^3 + x^2 + x + 1)(x^3 + x + 1)$

$= x^6 + x^5 + x^4 + x^4 + x^3 + x^2$
$+ x^3 + x^2 + x$

Here, duplicate bit's addition will result 0

So, $C(x) = x^6 + x^5 + x$

Hence, from the above Codeword polynomial, the codeword will be:-

$$C = [1\,1\,0\,0\,0\,1\,0]$$

Systematic cyclic Encoding

Message Signal, $M = [1011]$

polynomial $\longrightarrow$ $x^{3}\times 1 + x^{2}\times 0 + x^{1}\times 1 + x^{0}\times 1$

$= \cancel{x^4 + x^2 + x}$

$= x^3 + x + 1$

Generator polynomial, $G(x) = x^3 + x + 1$

The eqn for determining 7 bits codeword for systematic code is given as: $C(x) = x^{n-k} . M(x) + P(x)$

$\Rightarrow$ ভাজ্য = ভাজক × ভাগফল + ভাগশেষ

$P(x)$ represents the parity polynomial and is given by, $P(x) =$ Remainder of $\left\{ \dfrac{x^{n-k} . M(x)}{G(x)} \right\}$

Example:-

Construct a systematic Cyclic codes (7,4) using generator polynomials $g(x) = x^3+x+1$ with message (1011)

So

~~$x^3+x+1)$~~

Solve:-

Message code = 1011

in polynomial = $x^3 + x + 1$

~~$x^3+x+1)\ x^3$~~

Here,

x^{n-k} of $m(x) = x^{7-4} = x^3\ (x^3+x+1)$

$= x^6 + x^4 + x^3$

Now need to divide it with $g(x)$

$$x^3+x+1 \overline{)\ x^6+x^4+x^3\ (} \ x^3$$

$$x^6+x^4+x^3$$

$$\overline{\ 0\ }$$

g. $P(x) = 0$

i. Codeword polynomial equation,

$$C(x) = x^{n-k} M(x) + P(x)$$

$$= x^3 (x^3 + x + 1) + 0$$

$$= x^6 + x^4 + x^3$$

$\therefore$ Code polynomial, $c = [1011000]$

(Ans)

☐ Systematic cyclic Encoding:-

~~M(x)~~ =

Message signal : $M(x)$

Generator polynomial, $G(x)$

Systematic code, $C(x)$

parity polynomial, $P(x)$

$$P(x) = \text{Remainder of } \left(\frac{x^{n-k} \cdot M(x)}{G(x)} \right)$$

⊛ Consider, Message signal, $m = [\underbrace{1011}_{4}]$

So the message polynomial, =

$$M(x) = 1 + x^2 + x^3$$

Grenarator polynomial $g(x) = x^3 + x + 1$

~~We know that~~

Determine 7 bit Codeword For Systematic Code.

Solve:-

$$m(x) = (1 + x^2 + x^3)$$

$$G(x) = (x^3 + x + 1)$$

We know that,

7 bit Codeword = $x^{n-k} \cdot m(x) + P(x)$

We know that

$$P(x) = \text{Remainder of } \frac{x^{n-k}(m(x))}{G(x)}$$

Here,

~~$m(x) = 1 + x^2 + x^3$~~

$$(=) \text{ Remainder of } \frac{x^{7-4} \cdot m(x)}{G(x)}$$

$$= \frac{x^3(1 + x^2 + x^3)}{x^3 + x + 1}$$

$x^3+x+1 \overline{) \, x^3+x^5+x^6} \left(1+x^{[illegible]}+x^2+x^3 \right.$

x^3+x+1

$x+x^5+x^6+1$

$x+x^4+x^2+$ [illegible]

~~$x+x^6+x^3+x^2+1$~~

$x^5+x^4+x^6+x^2+1$

$x^5+ \quad x^2 \quad +x^3$

$x^4+x^6+1+x^3$

$x^4+x^6 \quad +x^3$

$1 \Rightarrow P(x)$

7-bit codeword = $x^3(1+x^2+x^3)+1$

$x^3(x^3+x^2+1)+1$ $= x^3+x^5+x^6+1$

$x^6+x^5+x^3+1$ $=$ ~~$1+x^3+x^5+x^6$~~ ~~x^6~~

$= 1101001$

$=$ ~~0100101~~

~~= 1001011~~ $= \boxed{1101001}$

এখানে ম্যাসেজ vector লেখার সময় বড় হতে ছোট

☐ Systematic Block Code :-

$U = (P_1, P_2, P_3, \ldots, P_{n-k-1}, m_0, m_1, \ldots m_{k-1})$

⊛ Generator polynomial; $G(x) = 1+x+x^3$

generator a systematic codeword for (7,4)

Codeword for the message vector, $m = 1011$

=)

Solution :-

$\therefore m = 1011$

$M(x) = 1+x^2+x^3$

$G(x) = 1+x+x^3$

Parity, $P = 7-4 = 3 = 1+x+x^2$

Divide x^{n-k} of $m(x)$ by $g(x)$ using polynomial

$x^3(1+x^2+x^3)$

$= x^3 + x^5 + x^6$

$= x^6 + x^5 + x^3$ $= 110100$

$1+x+x^3 \,\big)\, x^6 + x^5 + x^3 \,\big(\, 1 + x + x^2 + x^3$

$1 + x \quad x^3$

$x^6 + x^5 + 1 + x$

$x^2 + x^4 \quad + x$

$x^6 + x^5 + 1 + x^4 + x^2$

$x^5 + 1 \quad x^3 + x^2$

$x^6 + 1 + x^4 + x^3$ ~~[illegible]~~

$x^6 \quad x^4 + x^3$

1 ~~[illegible]~~

$\therefore$ ~~$Q(x)$~~ $=$ ~~[illegible]~~

$\therefore v(x) = P(x) + m(x)$

$= \underbrace{100}_{P}\underbrace{1011}_{m}$

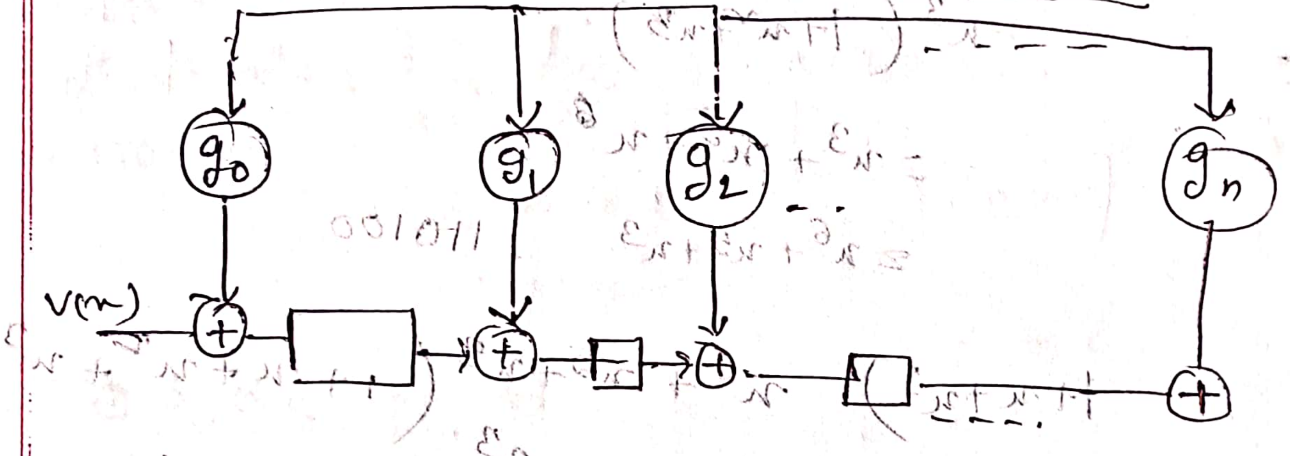
$1 \times x^0 + 1 \times x^{1} + 1 \times x^2$

$= 1001011$

~~1101001~~

Circuit Dividing polynomial

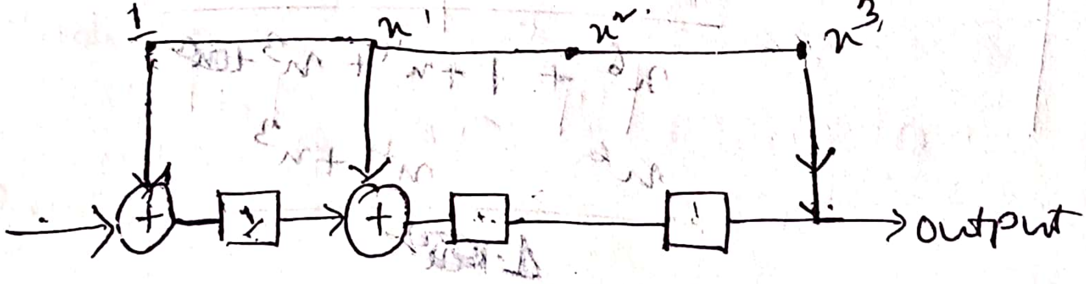
Model



⊛ $V(x) = x^6+x^5+x^3$
$g(x) = 1+x+x^3$ } so draw the circuit

Solve:-

$g(x) = 1+x+x^3$



Figure

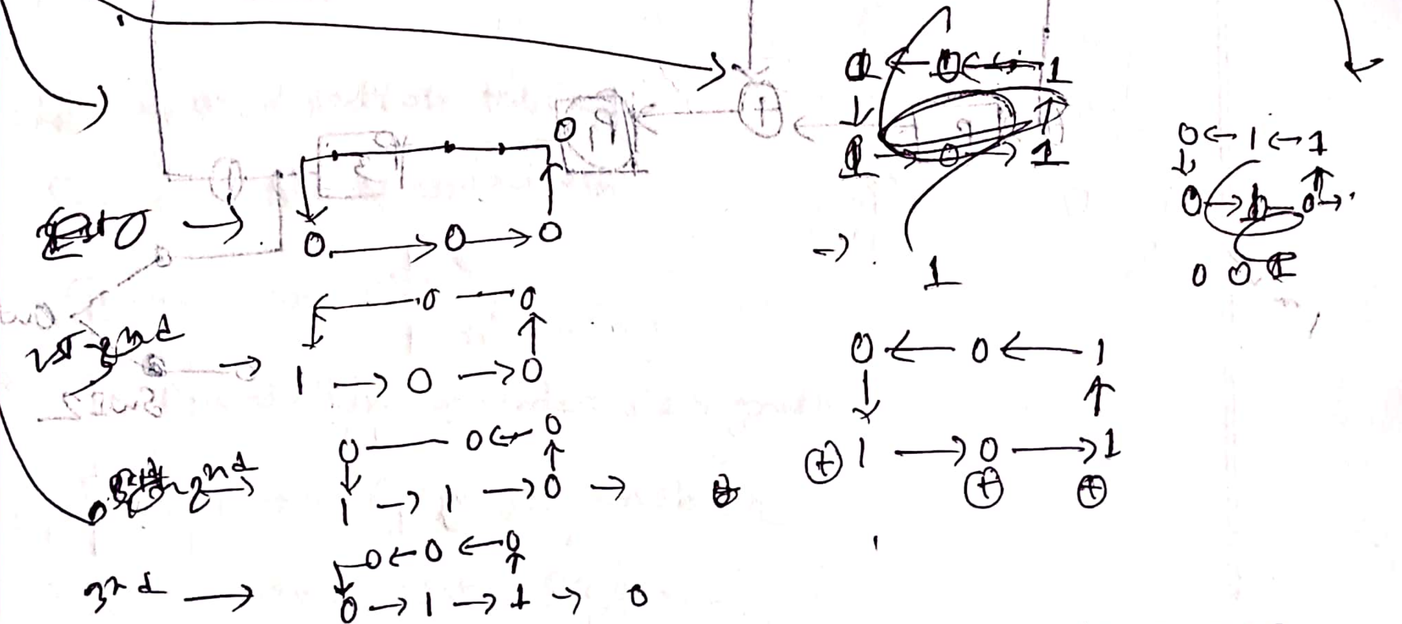
Here, $V(x) = x^6+x^5+x^3$

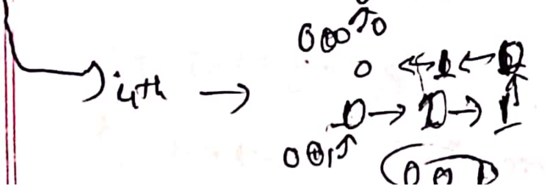
= ~~111000~~

= 0001011

$x^4 = 0$
$x^2 = 0$
$x = 0$
$1 = 0$

Input queue	Shift number	Register content	Output & Feedback
0001011	0	000	0
000101	1	100	0
00010	2	110	0
0001	3	011	0
000	4	~~001~~ 1 0 1 001 111	1
00	5	~~000~~ 001	1
0	6	0101	1
—	7	100	1



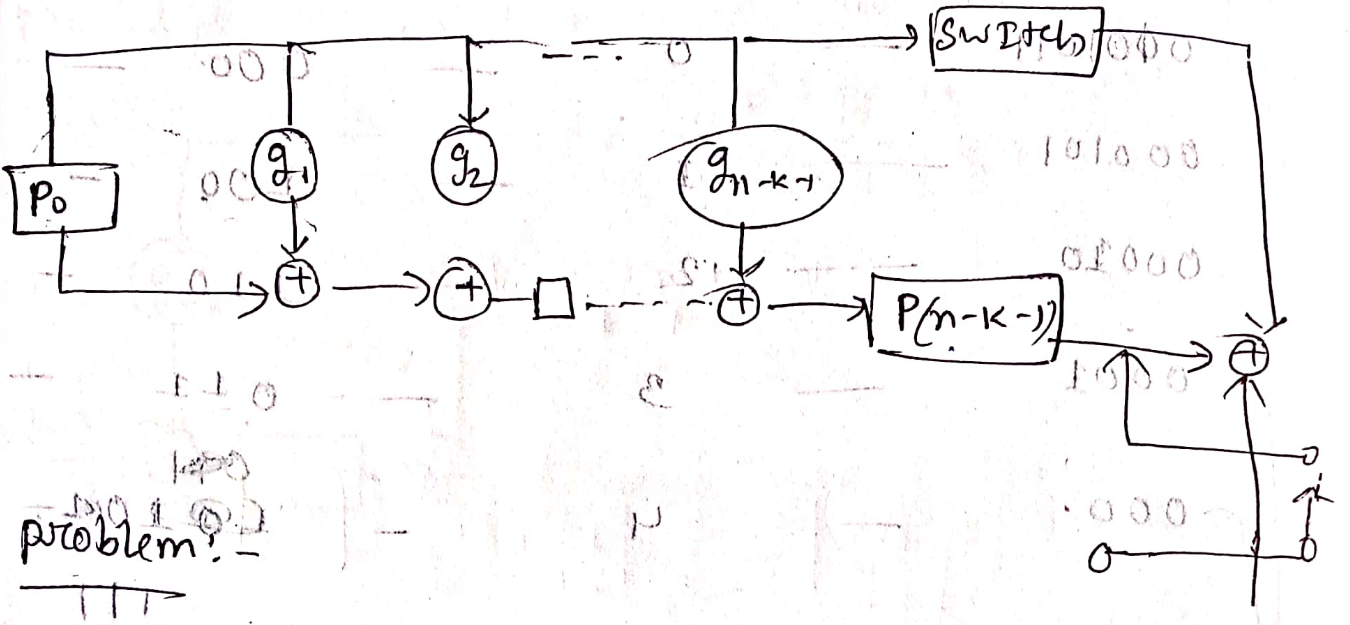


∴ codeword = ~~1000111000~~

= $P(x) + x^{n-k} \cdot m(x)$

= 1001111000

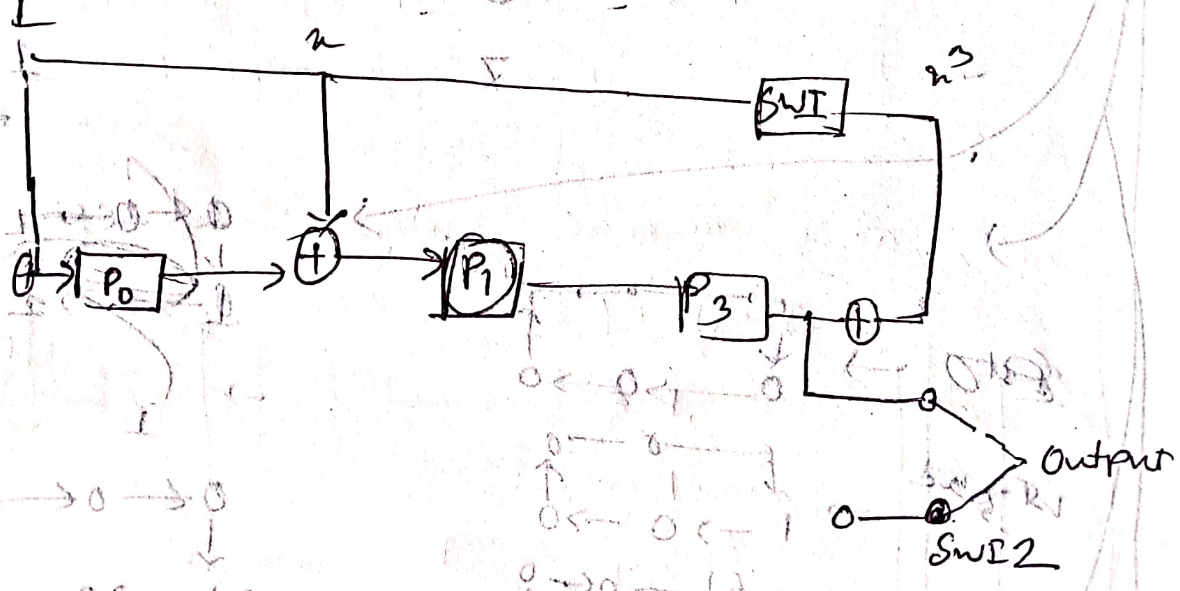
Codeword :- $P(x) + m(x)$



problem :-

(7,4) codeword $m = 1011$

$g(x) = 1 + x + x^3$



1 → 0 → 0   1 → 1 → 0

Input queue	Shifting number	Register content	Output
1011	0	000	—
101	1	110 100	1
10	2	101	1
1	3	101	0
—	4	100	1

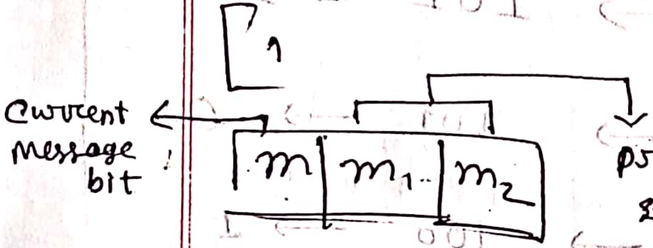
$$\text{Codeword} = P(x) + (x)^{n-k} \cdot m(x)$$

$$= 1001011$$

- ☐ error pattern table
- ☐ Location error pattern,
- ☐ Convolution Coding
- ☐ Convolution encoder designation
- ☐ Impulse Response encoder
- ☐ Encoder State Diagram
- ☐ Tellis diagram
- ☐ Mutual Information [Mid PLFC1]

Convolution Codes :-

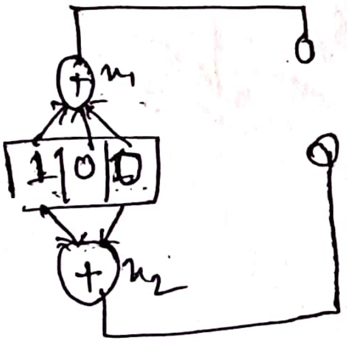
In convolution codes, block of 'n' code words generated by the encoder in time unit depends on not only block of 'K' message digits with in that time unit but also on the preceding (m-1) blocks of message bits



previous two successive message bit which represent state of convolution code

$m = 101$ → Here, Constant length = Number of register = 3

[এখানে নিয়ম হচ্ছে x_1 → বা উপরের ⊕ এর সাথে তিনটা বিটই add হবে। আর নিচের x_2 বা নিচের ⊕ " " First এবং Last bit add হবে]



$x_1 = \sout{m_1 \oplus m_2 \oplus}\, m \oplus m_1 \oplus m_2$

$= 1 \oplus 0 \oplus 0$

$= 1$

$x_2 = m \oplus m_2$

$= 1 \oplus 0$

$= 1$

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

⊛ Another type of error correcting code where the output bits are

⊛ P = n − k

Code rate, $r = \frac{k}{n}$

K = no. of ~~msg~~ msg bit,

n = no. of encode o/p bits

K = Constraint length

↓

Single message bit Influences encoder o/p for different successive shift.

⊛ Shifting alway

$2^3 = 8$ bit হতে 2 কম

$8 - 2 = 6$ বার shifting হবে।

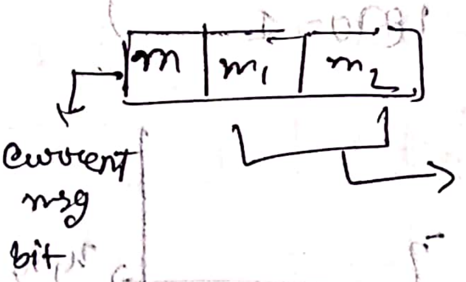
⊛ K = 3,

$K - 1 = 3 - 1 = 2$ টি zero দিয়ে register কে Flash দিব

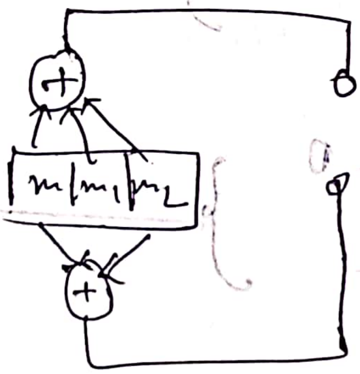
Code Dimensions: $(n,k) = (2,1)$

Convolution Code States
&
Code tree

⊛ Initially all registers are 0.



→ State of ~~cod~~ convolution Codes



$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

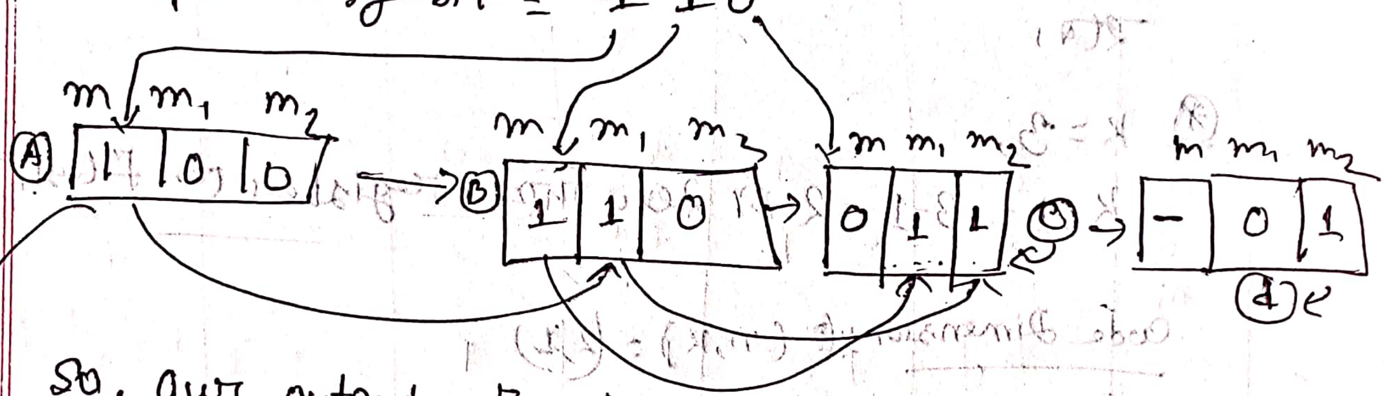
m_1 & m_2

→ Define the states

Ⓟ Code tree:-

→ each branch of tree represent an i/p symbol with corresponding pair of output binary symbols indicating on the branch.

⇒ Input msg bit = 1 1 0



So, our output ~~for~~ for convolution code,

Ⓐ, $x_1 = m \oplus m_1 \oplus m_2 = 1 \oplus 0 \oplus 0 = 1$

$x_2 = m \oplus m_2 = 1 \oplus 0 = 1$

output $x_1 x_2 = 11$

For Ⓑ,

$x_1 = 1 \oplus 1 \oplus 0 = 0$

$x_2 = 1 \oplus 0 = 1$

$x_1 x_2 = 01$

For c,

$x_1 = 0 \oplus 1 \oplus 1 = 0$

$x_2 = 0 \oplus 1 = 1$

$x_1 x_2 = 01$

Code Tree

we know,

m_1	m_2	State
0	0	a(00)
0	1	b(01)
1	0	c(10)
1	1	d(11)

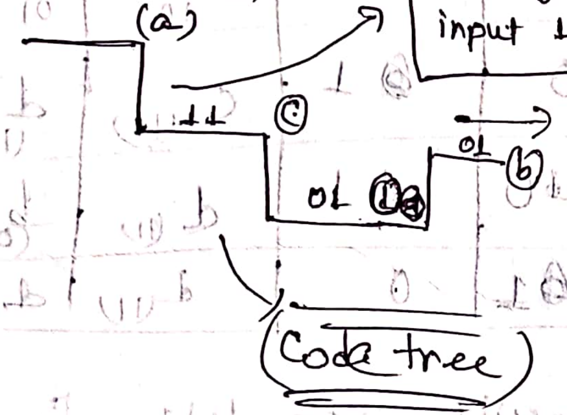
So, A have state-a as $m_1 = 0$ $m_2 = 0$

B " State-c as $m_1 = 1$ $m_2 = 0$

C " State-~~b~~d $m_1 = 1$ $m_2 = 1$

D " State-b $m_1 = 0$ $m_2 = ~~0~~1$

State-a



When you give input 1 goto down step

Note:-
While making it we
will see mbit only

→ Up step means input = 0

Down " " " = ~~0~~1

Code Tellis & State Diagram of Convolutional Codes

- First we need to calculate all possible state :-

m	m_1	m_2	x_1	x_2	Current state	Next state
0	0	0	0	0	a 00	a 00
1	0	0	1	1	a 00	c 10
0	0	1	1	1	b 01	a 00
1	0	1	0	0	b 01	c 10
0	1	0	1	0	c 10	b 01
1	1	0	0	1	c (10)	d (11)
0	1	1	0	1	d (11)	b (01)
1	1	1	1	0	d (11)	d (11)

- First write down all possible table for both 0 & 1.
- For current state use bits m_1 & m_2
- For next state count for m & m_1

$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

⊛ Then using this we will have to draw code triellis.

While drawing need to remember couple of things:-

⊛ Solid lines ("————") explains input bit 0

⊛ Dash lines ("- - - - - -") " " " 1

এটা বুঝলেই Trellis বুঝা হয়ে যাবে ~~State Diagram~~

Trellis Diagram

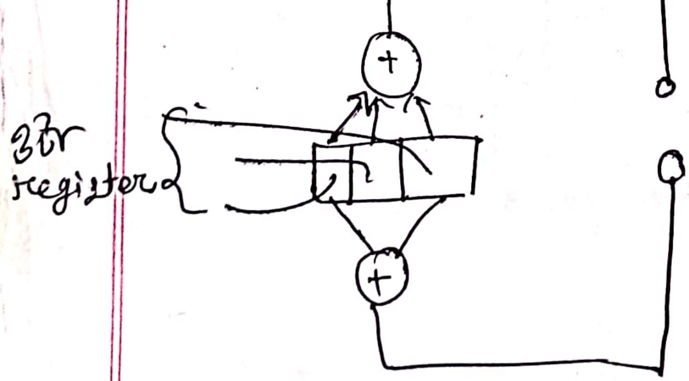
Example:-

For the encoder shown in figure shown in figure Show the state changes & the resulting output codeword sequence u for the message sequence m = 1 1 0 1 1 followed by, K − 1 = 2 zeros to flush the register. Assume that the initial contents of the registers are zeros;

Solve:-

$u_1 = m_1 \oplus m_2 \oplus m_3$

Figure



$u_2 = m_1 \oplus m_3$

$m = 1\,1\,0\,1\,1$

Current state =

O/P sequence সম্বন্ধে ধারণা

Input	Register contents	Current state at time t_i	Next state at time t_{i+1}	Branch word at time t_i	
				x_1	x_2
	0 0 0	0 0	0 0	0	0
1	1 0 0	00 a	10 c	1	1
1	1 1 0	10 c	11 d	0	1
0	0 1 1	11 d	01 b	0	1
1	1 0 1	01	10	0	0
1	1 1 0	10	11	0	1
0	0 1 1	11	01	0	1
0	0 0 1	01	00	1	1

001: Next state, current state

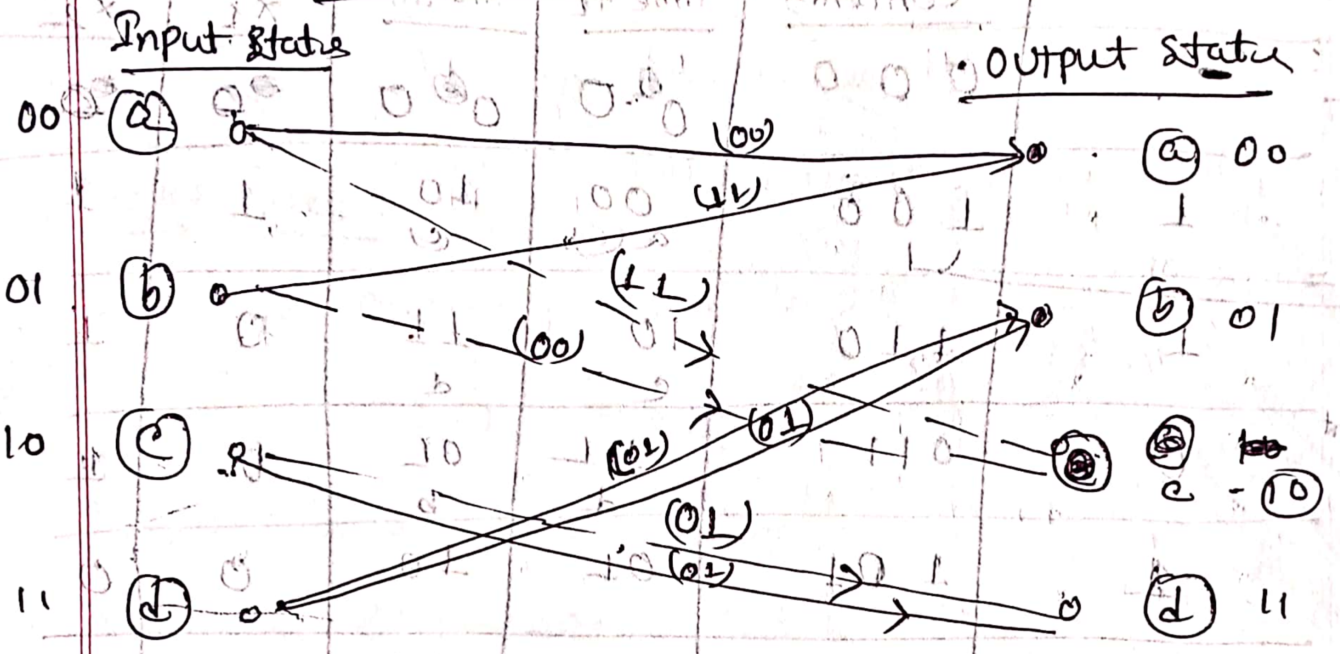
এই ২টি current stat

So the output sequence,

~~U= 11 10 11~~

U = 11 01 01 00 01 01 11

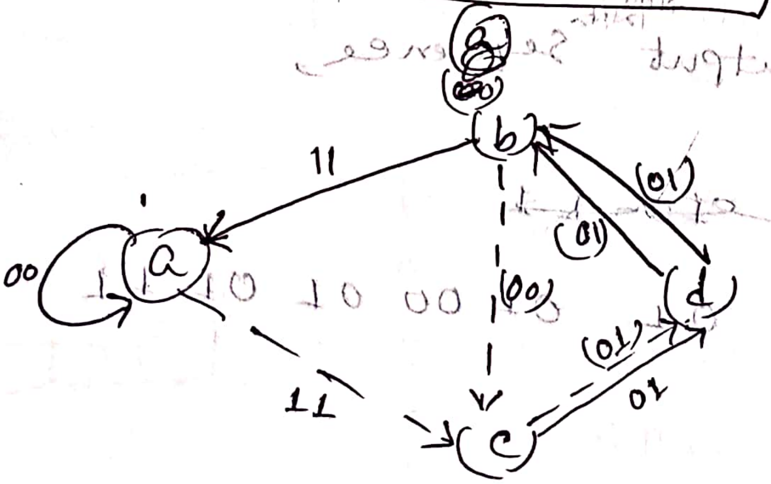
Code Trellis diagram



0→ ______ (Solid line)

1→ - - - - - - (dot line)

State Diagram



Viterbi Algorithm

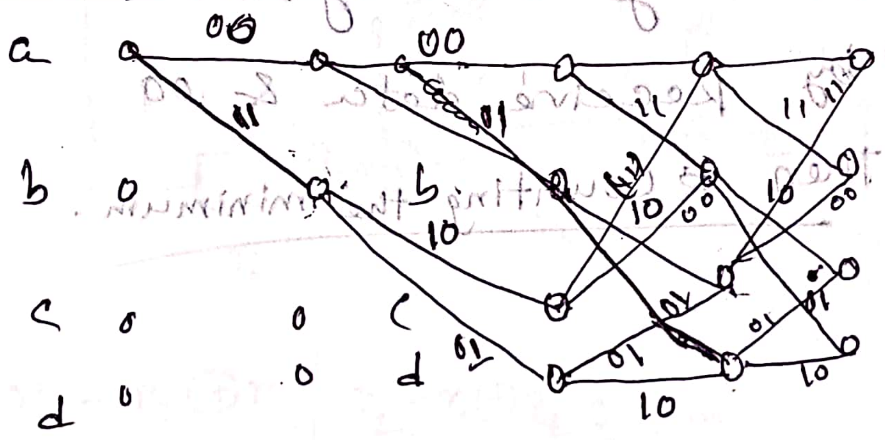
	m_0	m_1	m_2	x_1	x_2
a	0	0	0	0	0
	0	0	1	1	1
b	0	1	0	1	0
	0	1	1	0	1
c	1	0	0	1	1
	1	0	1	0	0
d	1	1	0	0	1
	1	1	1	1	0

Shows current state (m_0, m_1)

Show next state (m_1, m_2)

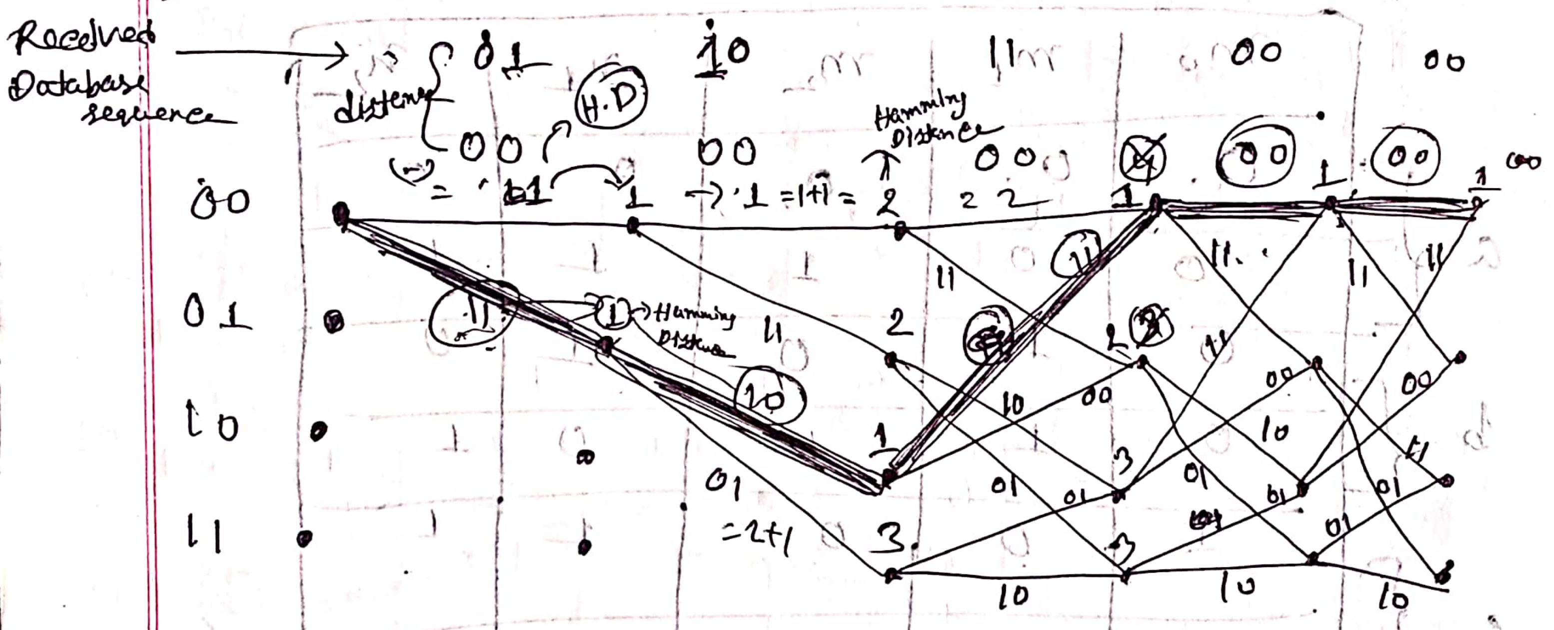
output (x_1, x_2)

Trellis:



Received Data Base sequence is given by,— 01 10 110000

Step ③ Match weighted with respect to Trellis diagram



So, Receive Data = 0110 110000

After Decoding,

Decoded Data = 11 101100 00

④ ~~Finding~~ -

Finding Hamming Distance by Receive data & 00 Then counting the minimum.

Ⓓ Encoder Circuit :-

Design the encoder for (7,4) cyclic code generated by $g(x) = x^3 + x^2 + 1$ & generate Codeword for $1100 = m$

$n = 7$, $k = 4$

$$\Rightarrow g(x) = 1 + \sum_{i=1}^{n-k-1} g_i x^i + x^{n-k}$$

$$g(x) = x^3 + x^2 + 1$$

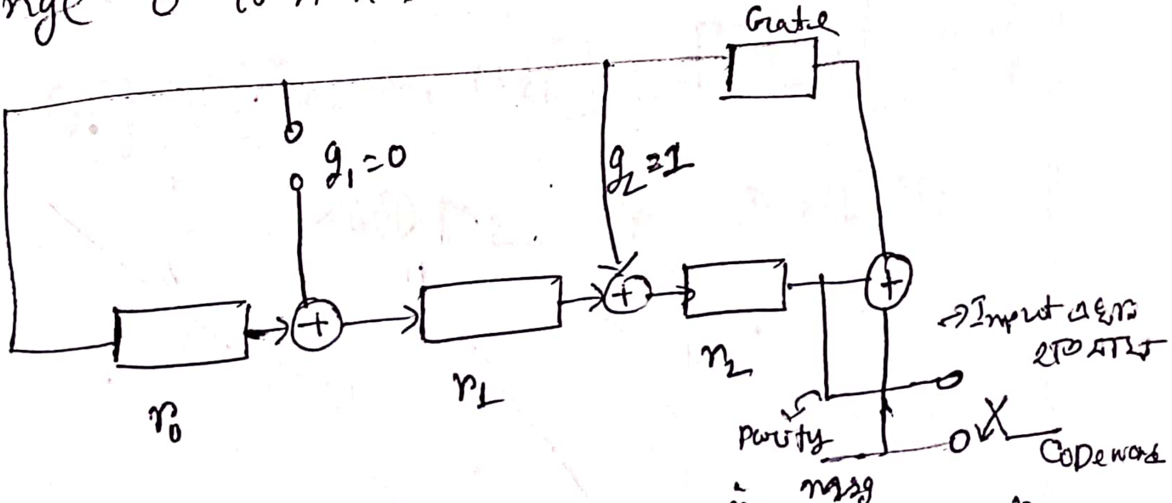
$$= x^3 + g_2 x^2 + g_1 x^1 + 1$$

$g_2 = 1$

$g_1 = 0 \rightarrow$ যেহেতু x $g(x)$ এ নেই

Number of Flip Flop $n - k = 7 - 4 = 3$

Range 0 to $n-k-1 = 0$ to 2



$r_0 r_1 r_2$
$= 0\,0\,0$

$r_0 = r_2 \oplus m$ | $r_2 = r_1 \oplus g_2 r_2 \oplus m$

$r_1' = r_0$

$P_1 = \underline{P_3} \oplus m$

$P_2 = P_1$

$P_3 = P_2 \oplus P_3 \oplus m$

previous value

$r_0' = \underset{\uparrow}{r_2} \oplus m$, $r_1' = r_0$

$r_2' = r_1 \oplus r_2 \oplus m$

$m = 1100$

Shift Number	Input (m)	Register Content r_0	r_1	r_2	Transmit bit (output) c
0		0	0	0	
1	1	1	0	~~0~~1	1
2	1	0	1	0	1
3	0	0	0	1	0
4	0	1	0	1	0
5		0	1	0	1
6		0	0	1	0
7		0	0	0	1

কারন। সব bit transmit হলে Switch বন্ধ হয়ে যাবে

$[P_K \; I_{n-K}]$

(6,3)

$6-3$

$$G = \left| \begin{matrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ \underbrace{0 & 0 & 1}_{I_K} & \underbrace{1 & 1 & 0}_{P} \end{matrix} \right|$$

$[C] = [M] [P]$

$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Code bit, $C = 6-3 = 3$

$\therefore$ ~~$A=8$~~ 3 Here, $M = K = 3$

Now,

$[C] = [M] [P]$

$$[c_0 \; c_1 \; c_2] = [M_0 \; M_1 \; M_2] \quad [P] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= M_0 \oplus M_2 \quad M_1 \quad M_0 \oplus M_1$$

Co

M_0	M_1	M_2	C_0	C_1	C_2	
0	0	0	0	0	0	→ 0
0	0	1	1	0	0	→ 2
0	1	0	0	1	1	→ 3
0	1	1	1	1	1	→ 5
1	0	0	1	0	1	→ 3
1	0	1	0	0	1	→ 3
1	1	0	1	1	0	→ 4
1	1	1	0	1	0	→ 4

$d_{min} = 2$

$$t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor$$

$$= \left\lfloor \frac{2-1}{2} \right\rfloor$$

$0.5 \leq t$

Ceil

$= \frac{1}{2}$

$= 1$

$$H = \begin{bmatrix} P^T & I_{n+k} \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans

1101101

~~1100~~11 = R

$$S = R \cdot H^T$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 1 \oplus 1$$